A Model of the Evolution of Executive Labor Markets*

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Abstract

We present a model of the market for executives that is consistent with many empirical facts about executive compensation, pay inequality, turnover, and mobility across firms. A novel feature of our model is the existence of two different equilibria in executive markets. This feature fits well with the existing evidence that executive labor markets have become structurally different in more recent times, as the empirical literature suggests. The model allows us to provide a unified narrative of the evolution of executive labor markets.

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1. Introduction

We present a model of executive labor markets that is consistent with many empirical facts about executive compensation, pay inequality, turnover, and mobility across firms. A novel feature of our model is the existence of two different equilibria in executive markets. In the first equilibrium, which we call *the old economy*, executive mobility is nonexistent, the link between firm size and executive compensation is weak, pay inequality among executives is low, and executive turnover in large firms is low. In contrast, in the second equilibrium, which we call *the new economy*, we observe a high degree of mobility, a strong link between firm size and compensation, high levels of pay inequality, and high turnover at the largest firms.

Our choice of labels is not accidental. Our old economy equilibrium resembles the market for executives in the US from the 1940s to mid 1970s (Frydman, 2005; Frydman and Saks, 2010; Jenter and Frydman, 2010). Our new economy equilibrium has characteristics that are normally associated with the market for executives since the mid 1970s. Our parsimonious framework provides a unified narrative of the evolution of the market for executives. As the equilibrium is always in unique for a given set of parameters, we can study how changes in regimes occur. The model explains a number of puzzling empirical facts, has many testable predictions, and has social welfare implications that are relatively easy to analyze.

Our analysis builds upon some of the existing theoretical ideas on executive markets. As in firm-CEO matching models, managers and firms are heterogeneous (Gabaix and Landier, 2008; Tervio, 2008; Eisfeldt and Kuhnen, 2011). As in Murphy and Zabojnik (2006) and Frydman (2005), managers are endowed with both firm-specific and general skills. Thus, as in these papers, exogenous changes in the relative importance of firm-specific skills lead to changes in executive turnover and mobility across firms. As in many models that emphasize incentive problems inside firms (e.g. Edmans, Gabaix, and Landier, 2009, and Edmans and Gabaix, 2011), we also consider incentive compensation as a means to alleviate moral hazard problems. We use these ideas as building blocks, which we combine to arrive at new predictions.

A unique feature of our analysis is the possibility of drastic changes in the nature of the market for executives as a consequence of small changes in the underlying parameters in the economy. We show that there are a number of triggers of structural changes in the market for executives. Some previous papers have emphasized the growing importance of general managerial skills relative to firm or industry-specific skills as one of these triggers (Murphy and Zabojnik, 2006; Frydman, 2005). Our model has similar predictions. Additionally, we uncover some other triggers that have been overlooked by the literature. We show that, as firms become more heterogeneous, eventually a change from the old regime to the new regime must occur. Thus, a gradual increase in firm size inequality is sufficient to explain a quick transition from an equilibrium with low executive mobility and low pay to an equilibrium with high mobility and high pay. We also show that an increase in the death rate of firms (which, for example, could be driven by tougher product market competition) could lead to a transition to the new equilibrium.

Our model provides an explanation for the evidence on the differences in executive mobility and compensation in older and more recent times. For example, the model quite naturally generates a weak link between firm size and executive pay in the old regime, and a strong link between size and pay in the new regime. The difference between older and more recent times is one of the main puzzling findings documented by Frydman and Saks (2010). To explain these differences, we only need to assume that either the dispersion in firm profitability or the importance of general skills increase continuously over time.

As is the case with any equilibrium model, the intuition for the results is never perfectly clear without first working through the details of the model. In this introduction, we give only a partial and informal description of the results; later in the paper, we explain the arguments in more detail.

We assume that some firms are inherently more profitable than others. More profitable firms endogenously choose to become larger than less profitable firms. Thus, for simplicity of exposition, we call the set of more profitable firms "large firms," and the less profitable firms "small firms," with the implicit understanding that size is endogenously determined. If all else is kept constant, high-talent managers should work for large firms, because managerial talent and firm productivity are complements. However, if a manager that currently works for a small firm moves to a larger firm, her productivity falls due to a loss of firm-specific skills. Thus, for executive mobility to occur in equilibrium, the profitability differential between small and large firms must be sufficiently large. Alternatively, for a given gap in profitability, as firm-specific skills become less important, executive mobility eventually becomes possible.

Consider now the case of a small firm and its incumbent manager. As time passes, the firm (i.e. shareholders or the board of directors) privately learns about its manager's ability. It then chooses whether to retain the manager or to hire (or promote internally) a new manager. Because large firms observe the retention and firing decisions of small firms, large firms rationally infer that managers who are retained by small firms have above-average abilities. A large firm that is unhappy with the ability of its current manager would want to poach a manager that is retained by a small firm. When faced with a poaching attempt, the small firm has two choices: it may choose to fight, and thus match the offer that the large firm makes to the manager, or it may choose to let the manager go without a fight. These two alternatives give rise to two different equilibria or regimes. In the old economy equilibrium, a small firm always fights poaching attempts, which implies that no actual poaching occurs in equilibrium. In this case, there is no managerial mobility across firms. Consequently, the quality of the match between firms and manages is poor; high-ability managers only end up in large firms by chance. Therefore, in the old economy equilibrium there is no clear link between firm size and managerial talent, and there is only a tenuous link between firm size and executive pay.

In the new economy equilibrium, a small firm never fights a poaching attempt. Thus, in equilibrium, large firms often poach the best managers from small firms. Executive mobility is substantial, which implies a better matching between firm size and managerial talent than the one that is observed in the old economy. If high-ability managers are short supply, the link between compensation and firm size also becomes stronger.

The new economy equilibrium is more efficient than the old economy equilibrium in a number of aspects. Compared to the old economy, in the new economy the quality of the manager-firm match is improved and inefficient turnover is reduced. A change from the old economy to the new economy also has distributional consequences. In the new economy, provided talent is scarce, there is more pay inequality between top executives and other executives.

We now turn to the description of the model. We postpone to the end of the paper a detailed discussion of the related theoretical and empirical literature.[1]

2. Model Setup

2.1. Firms and Managers

We consider an infinite-horizon economy populated by a continuum of infinitely-lived firms. Firms are risk-neutral and share a common discount factor $\delta \in [0, 1)$ (we use "firms" as a shortcut for either shareholders or boards of directors that are fully aligned with shareholders). Each firm is uniquely identified by a real number j. Firms are of two types: high-profitability (type $i = h$) or low-profitability (type $i = l$). The mass of h-firms is H and the mass of l-firms is L. A type-i firm is endowed with a technology represented by a

[1] This is not yet in this version.

productivity parameter $i \in \{l, h\}$. We set $h = \theta > 1 = l$. That is, the productivity parameter of an l-firm is normalized to one, therefore θ can also be interpreted as the productivity advantage of h-firms relative to l-firms (or the productivity differential, for short).

All firms need a manager (e.g. a CEO) to operate their technologies. Managers live for two periods: young age and old age. At each period t ($t = 0, 1, 2, ...$) a mass M of young managers enter the labor market. Young managers are in excess supply: $M > H + L$. For consistency, we also assume that a mass M of old managers exists at time $t = 0$.

Managers are risk-neutral and have an outside option of working as non-executives for an exogenously given compensation, which we normalize to zero. We assume that managers are protected by limited liability; they must be paid a non-negative (per period) salary w. Young managers have zero personal wealth. These assumptions simplify the analysis, as they rule out cases in which young managers pay fees to work, and cases in which old managers pay fines if they quit from their firms. These assumptions jointly imply that firms can only offer one-period compensation contracts to managers.

Managers are endowed with some general managerial skills, which are represented by a real number $\tau \in [\underline{\tau}, \overline{\tau}]$, where $\underline{\tau} \geq 0$ and $\overline{\tau} < \infty$. We refer to τ as a manager's *general talent*, or simply as *talent*. Young managers enter the labor market with unknown general talent. All managers and firms know that the talent τ of a young manager is drawn from a cumulative distribution function $F(\tau)$ on the bounded support $[\underline{\tau}, \overline{\tau}]$. For tractability, we assume that F is twice differentiable.

If a firm hires a young manager at the beginning of period t, both the firm and the manager expect the manager's talent to be $\mu \equiv \int_{[\underline{\tau}, \overline{\tau}]} \tau dF(\tau)$. If the same (now old) manager is still employed by the firm at the beginning of period $t + 1$, both the firm and the manager learn τ. One interpretation is that knowledge of τ is a by-product of the relationship between the firm and the manager. Consistent with this interpretation, we assume asymmetric learning, in the sense that only the firm and its incumbent manager learn the latter's type. Everyone else remains uninformed about τ.

The technology requires two managerial inputs: general talent and firm-specific skills. In an extension of our model (in Subsection 6.2), we also consider effort as a third input. Firm-specific skill is measured by $s \in \{\gamma, 1\}$. A manager who has been working for a given firm has firm-specific skill $s = 1$ in her own firm, but would have firm-specific skill $s = \gamma \in (0, 1)$ if she was to move to a different firm.

The acquisition of firm-specific skills also requires one unit of time. If a firm hires a new manager (young or old) in period t, this manager has a firm-specific skill parameter of γ. If a young manager stays with the same firm until her old age, she then acquires firm-specific skills of $1 > \gamma$.

Production is stochastic and (conditional on the characteristics of the firm and of the manager) independently distributed across firms and across time. There are only two possible revenue states: *high*, in which the firm receives one unit of the numeraire, and *low*, in which the firm receives zero. At time t, if firm j, which has type i_j, employs a manager with firm-specific skill s_{jt} and general talent τ_{jt}, its period-t revenue is given by

$$R_{jt}(i_j, s_{jt}, \tau_{jt}) = \begin{cases} 1, & \text{with probability } \min\{i_j s_{jt} \tau_{jt}, 1\} \\ 0 & \text{with probability } 1 - \min\{i_j s_{jt} \tau_{jt}, 1\} \end{cases} . \tag{1}$$

For simplicity of exposition, we assume that we are always in the region where $i_j s_{jt} \tau_{jt} < 1$, thus we can ignore the *min* operator above. Under this assumption, the expected output of a firm endowed with(i_j, s_{jt}, τ_{jt}) is simply $i_j s_{jt} \tau_{jt}$. When there is no risk of confusion, we drop the subscripts j and t to simplify the notation.

This probabilistic interpretation places some restrictions on the parameters. First, we need $\overline{\tau} < 1$. Second, we also require $\theta \leq \overline{\theta} \equiv \frac{1}{\overline{\tau}}$. These assumptions can be seen as simple normalizations; they have no empirical implications.

A simplifying assumption that we adopt for most of this paper is that past performance offers no information about τ. That is, we assume that knowledge of R does not help in predicting τ. This assumption is not crucially important, and we adopt it only for simplicity. We later discuss the consequences of relaxing this assumption for our conclusions.[2]

2.2. Technical Assumptions

We have almost all the elements that we need to characterize the set of equilibria in our model. Before we do so, here we discuss some restrictions that we impose on the function F to simplify the analysis.

Firms would never replace an incumbent manager if the importance of firm-specific skills was too high. To focus on the interesting cases, we impose the following restriction on the parameters:

Assumption T1 $\underline{\tau} < \gamma\mu$.

Assumption T1 is not a very restrictive one, and it is trivially satisfied if, for example, we make $\underline{\tau} = 0$.

[2] This discussion is still missing from this preliminary version.

Define

$$E[\tau \mid \tau \geq \tilde{\tau}] \equiv \frac{\int_{\tilde{\tau}}^{\overline{\tau}} \tau dF(\tau)}{1 - F(\tilde{\tau})}. \tag{2}$$

We assume the following:

Assumption T2 $\frac{\partial E[\tau|\tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq \overline{\tau}$.

Assumption T2 is not necessary for any of the important results in this paper. It is however a convenient assumption that simplifies some of the proofs and guarantees equilibrium uniqueness in most of the cases that we consider.

Although we can prove most of the results without T2 (at the cost of more complexity), we also is note that T2 is not too restrictive. For example, any uniform distribution with $\overline{\tau} \geq \frac{1}{2}$ trivially satisfies Assumption T2.[3] We also do not need T2 to hold for all $\tilde{\tau} \in [\underline{\tau}, \overline{\tau}]$; most of our results can be proven for *any* differentiable F as long as $\overline{\tau} \geq \frac{1}{2}$, for a large set of parameters:

Lemma 1 *For any twice-differentiable F such that $\overline{\tau} \geq \frac{1}{2}$, there exists a τ_F such that condition T2 holds for all $\tilde{\tau} \in [\tau_F, \overline{\tau}]$.*

Proof. See the Appendix. ■

Finally we note that Assumption T2 is never a necessary condition for equilibrium existence or uniqueness. Equilibrium existence is trivially guaranteed for any continuous F. Assumption T2 is sufficient (but not necessary) for uniqueness.

2.3. Timing

We now describe the sequence of dates within each period t.

Date 1. At the beginning of each period, firm j is in one of two states: either the firm has an incumbent manager with talent τ or it has no incumbent manager. In the former case, the firm either makes an offer w to its incumbent manager or chooses to fire the incumbent manager. In the case in which the firm has no incumbent manager, no action is taken at this date.

Date 2. If firm j does not have an incumbent manager at this date, it chooses either to make an offer $\hat{w}^y$ to a young manager or to make an offer $\hat{w}_k$ to an old manager currently

[3] We note that T2 is similar to assuming log-concavity of F, which implies $\frac{\partial E[\tau|\tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \leq 1$. In fact, most of our results can be proven with log-concavity alone. Log-concavity is not very restrictive. Many families of distributions, including the Uniform distribution and (truncated) Normal distributions, are log-concave.

employed by firm $k \neq j$. In the case in which the firm has an incumbent manager, no action is taken at this date.

Date 3. Offer-holding managers initiate a bargaining game with their current employers. We describe the details of this game in the next subsection.

Date 4. The bargaining game ends, contracts are signed, and revenue is realized.

2.4. Mechanics of Competition

In this section we impose some structure on the operation of executive labor markets.

Consider a given manager-firm match. Suppose that the firm is satisfied with this match. This match can be contested in two ways. First, the incumbent manager may search for a better job. Second, a competing firm may try to poach the manager by offering her a better compensation contract. Thus, a bidding contest for the manager may be initiated by either *searchers* (managers looking for better jobs) or *poachers* (firms looking for better managers). For concreteness, we assume that all bidding contests are initiated by poachers, although this assumption is clearly inconsequential.

When a firm is targeted by a poacher, it has two options: it chooses either to *fight* (make counter-offers to its manager) or to let the manager go. Before describing the competition mechanism in detail, we make two initial assumptions:

Assumption C1 *A firm cannot commit not to fight for a manager.*

This assumption requires that firms always make offers and counter-offers whenever it is rational to do so. In particular, if a firm knows that it can win a fight for a manager, it cannot commit not to start a fight.

Assumption C2 *A firm will not enter a fight knowing that it cannot win.*

If a firm knows that there is no chance that it will win a fight, then it will not try to poach a manager and it will not react to a poaching attempt targeted at its manager. This assumption is not strictly necessary (that is, it does not affect the selection of equilibria), but it simplifies the analysis. Intuitively, fighting for a manager by means of a sequence of offers and counter-offers is costly (although we abstract from these costs in our model). Thus, firms would save on these costs if they do not enter a fight when they know that there is no chance that they might win.

We now describe the competition mechanism. A bargaining game between a manager and its current employer is initiated only if, at date 2, a poacher offers an old manager a salary $\hat{w}$ that is (infinitesimally) strictly better than w, the salary offer that the manager currently holds. The bidding contest occurs in the form of alternating binding offers, as in Stahl-Rubinstein bargaining games: after the manager receives an offer, her firm may make a counter-offer, which could then be matched by another counter-offer by the poacher, and so on. Rounds of offers and counter-offers occur sequentially, in continuous time, from initial round $\underline{r} = 0$ to $\overline{r} = 1$ (that is, during the one discrete-unit of time between dates 3 and 4). All offers and counter-offers must be sequentially rational. The bargaining game ends either when the employer does not make a counter-offer or when the poacher does not make a counter-offer. The manager then accepts the last offer standing.

At each bargaining round $r \in [0, 1]$, the negotiation between the manager and her current employer may break down for exogenous reasons, in which case the manager accepts the most recent offer by the poacher. One interpretation is that, at each round of offers, the relationship between the manager and the employer may become sour. We assume that the hazard rate $\lambda > 0$ - the probability of a negotiation breakdown at round $r + \epsilon$, conditional on arriving at round y and $\epsilon > 0$ very small - is constant.

The possibility of a negotiation breakdown implies that sequentially fighting for a manager is costly. All else constant, the employer would like to stop bargaining immediately. Thus, the exogenous probability of breakdown plays a similar role as time discounting in Stahl-Rubinstein games of bargaining.

At the end of the game, the losing firm immediately hires a young manager.

The particular structure of the competition described in this section is just one possible microfoundation for the selection of equilibria in our model. We could alternatively adopt an axiomatic approach, in which the bargaining protocol is left relatively unconstrained, as long as a small set of assumptions are satisfied. Such an axiomatic approach is more general but also more abstract.

3. Equilibrium Characterization

To build intuition, we first solve the model in the special case in which $\delta = 0$, that is, assuming that firms only care about current profits. This case is cleaner and easier to follow, and delivers most of the crucial results of our model. Focusing on this case significantly improves the clarity of the exposition. Later in the paper (in Subsection 6.3), we analyze the general case in which $\delta \geq 0$.

We note that the case in which $\delta = 0$ is not exactly identical to a fully static model. For example, in a static, one-period version of our model, the equilibrium managerial turnover rate is indeterminate. This is so because the number of firms that start a period without an incumbent manager is a free parameter in a one-period model. In the dynamic case (regardless of the value of δ), however, the proportion of firms without a manager at the beginning of each period is uniquely determined by how the game was played in the preceding period.

We only focus on stationary equilibria, that is, equilibria in which the equilibrium strategy profile is the same in every period. Stationarity is an intuitive requirement for equilibrium selection in our analysis, because we want to characterize the long-run properties of the executive labor market. Thus, changes in the nature of stationary equilibria can be interpreted as structural, lasting changes to the nature of this market.

3.1. No-poaching Equilibrium

In this section, we define, characterize, and prove the existence of the first of the two possible types of equilibrium in our model: an equilibrium in which managers do not move from one firm to another. We call this equilibrium a *no-poaching equilibrium.*

3.2. Definition

Definition 1 ***(No-poaching Equilibrium)*** *A no-poaching (stationary) equilibrium is a (time-invariant) set of salaries, retention thresholds, and turnover rates* $\{(\tilde{\tau}_l, w_l, \beta_l), (\tilde{\tau}_h, w_h, \beta_h)\}$ *such that:*

- *A firm of type* $i \in \{l, h\}$ *retains any incumbent manager with talent above* $\tilde{\tau}_i$ *and offers her a salary* w_i*;*
- *A firm of type* $i \in \{l, h\}$ *dismisses any incumbent manager with talent below* $\tilde{\tau}_i$*, in which case it hires a young manager, who accepts to work for a salary of zero;*
- *The salary* w_i *is such that no firm expects to gain from offering a higher salary to a manager from a different firm;*
- *The fraction of type-i firms that hire young managers (the turnover rate) is* $\beta_i = \frac{1}{2-F(\tilde{\tau}_i)}$*,* $i = l, h$*;*
- *All players know* $\{(\tilde{\tau}_l, w_l, \beta_l), (\tilde{\tau}_h, w_h, \beta_h)\}$*.*

A few comments on this definition are necessary. First, in this definition we already use the fact that any equilibrium in this game (with or without poaching) must have a *threshold property*, in which all managers whose talents are revealed to be above a given threshold are retained, while those below are fired. That this threshold property must hold is easy to see: for a given w_i, the expected profit from retaining a manager with talent τ is $\tau - w_i$. Thus, if it is worth retaining a manager with talent $\tilde{\tau}$, then it is worth retaining any manager with talent $\tau \geq \tilde{\tau}$.

Second, in this definition we restrict ourselves to *symmetric equilibria*, in which all firms of a given type i choose the same strategies. It is possible to show that non-trivial asymmetric equilibria may only exist for a constellation of parameters that has measure zero. We thus ignore these non-generic cases. The possibility of asymmetric equilibria still remains for a strictly positive mass of parameters, but the asymmetry in these equilibria is *trivial*, in the following sense. In a trivial asymmetric equilibrium, all firms of a given type behave exactly in the same way, except when they observe a realization of $\tau = \tilde{\tau}$. We call such an asymmetry trivial because an exact realization of talent $\tilde{\tau}$ occurs with probability zero. Thus we also ignore such cases.

Third, the assumption that there exist many young managers ($M > L + H$) imply that young managers have no bargaining power. Thus, young managers are paid their reservation salary, which we have normalized to zero for simplicity.

Finally, this definition uses the fact that stationarity imposes aggregate constraints on the turnover rate. For firms of type i, at the end of period t a fraction of the managers are retained (α_i) and a fraction of the managers are replaced by new hires (β_i). To guarantee that the equilibrium is stationary, we need these fractions to be the same at the end of period $t + 1$. This requirement implies the following:

	t	$t+1$
retained	α_i	$\beta_i(1 - F(\tilde{\tau}_i))$
new hires	β_i	$\alpha_i + \beta_i F(\tilde{\tau}_i)$

, (3)

for $i \in \{l, h\}$. The stationarity conditions then write:

$$\begin{cases} \alpha_i = \beta_i(1 - F(\tilde{\tau}_i)) \\ \beta_i = \alpha_i + \beta_i F(\tilde{\tau}_i) \\ \alpha_i + \beta_i = 1 \end{cases}, \tag{4}$$

which then imply $\beta_i = \frac{1}{2-F(\tilde{\tau}_i)}$. Notice that β_l and β_h correspond to the steady state rate of turnover in type-l and type-h firms, respectively.

3.3. Existence

We now describe the derivation of a no-poaching equilibrium. A firm of type i that retains an old manager with talent τ_i and pays a salary w_i has an expected profit of

$$\pi_i^o(\tau_i) = i\tau_i - w_i, \tag{5}$$

where $i \in \{l = 1, h = \theta\}$. As discussed above, if the firm chooses instead to hire a young manager, it offers $w^y = 0$. Thus, the profit from replacing the old manager with a young manager is

$$\pi_i^y = i\gamma\mu. \tag{6}$$

Note that this profit takes into account the loss in firm-specific skills, which is represented here by the parameter $\gamma < 1$.

Under no poaching, firm i's retention decision is given by comparing $\pi_i^o(\tau_i)$ with π_i^y, which leads to the following rule:

$$\text{If } i\tau_i - w_i \geq i\gamma\mu, \text{ retain the manager,} \tag{7}$$

$$\text{if } i\tau_i - w_i < i\gamma\mu, \text{ fire the manager.} \tag{8}$$

This rule implies a threshold decision (given w_i) to retain the manager only if $\tau_i \geq \tilde{\tau}_i$, where

$$\tilde{\tau}_i = \gamma\mu + \frac{w_i}{i}, \;\; i = l, h. \tag{9}$$

This condition links the equilibrium thresholds to the offers that are made to the incumbent managers.

We need another condition to pin down both w_i and $\tilde{\tau}_i$. This is the no-poaching condition. To derive this condition, suppose that a manager retained by a firm of type i, $i = l, h$, receives an offer from a type-h firm. Here we only need to consider the cases in which the potential poachers are type-h firms, because if a poaching attempt is profitable for a type-l firm, it is also profitable for a type-h firm. The minimum offer that the manager would accept is w_i. Assuming that the minimum offer is made, if the poaching attempt is successful, firm h's profit is

$$\hat{\pi}_h(\tilde{\tau}_i) = \theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_i] - w_i. \tag{10}$$

In a no-poaching equilibrium, this profit must not be larger than the one obtained by hiring a young manager. Thus, we have:

$$\theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_i] - w_i \leq \theta\gamma\mu. \tag{11}$$

The condition above cannot hold with slack in equilibrium, because if it did, firm i could reduce w_i and still avoid the poaching of its manager. Thus, in a no-poaching equilibrium, condition (11) must hold with equality. We have that:

$$w_i = \theta\gamma\left(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu\right), \tag{12}$$

which, together with condition (9) yields

$$i\left(\tilde{\tau}_i - \gamma\mu\right) = \theta\gamma\left(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu\right) \tag{13}$$

which determines the equilibrium pair $(\tilde{\tau}_l, \tilde{\tau}_h)$.

Proposition 1 ***(Existence of no-poaching equilibrium)*** *A no-poaching equilibrium exists and is unique if and only if* $\theta \leq \min\left\{\theta^*, \overline{\theta}\right\}$, *where*

$$\theta^* \equiv \frac{\overline{\tau} - \gamma\mu}{\gamma(\overline{\tau} - \mu)}. \tag{14}$$

Proof. *If.* The threshold for an h-firm is given by

$$\tilde{\tau}_h - \gamma E[\tau \mid \tau \geq \tilde{\tau}_h] = 0. \tag{15}$$

For $\tilde{\tau}_h = \underline{\tau}$, the left-hand side of (15) becomes $\underline{\tau} - \gamma\mu < 0$ (from T1). For $\tilde{\tau}_h = \overline{\tau}$, the left-hand side of (15) becomes $\overline{\tau} - \gamma\overline{\tau} > 0$. Therefore a solution exists. The solution is unique if $1 - \gamma\frac{\partial E[\tau|\tau\geq\tilde{\tau}_h]}{\partial\tilde{\tau}_h} > 0$, which is implied by T2.

The threshold for an l-firm is given by

$$\tilde{\tau}_l - \theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_l] + \gamma\mu(\theta - 1) = 0. \tag{16}$$

For $\tilde{\tau}_l = \underline{\tau}$, the left-hand side of (16) becomes $\underline{\tau} - \gamma\mu < 0$. For $\tilde{\tau}_l = \overline{\tau}$, the left-hand side of

(16) becomes

$$\overline{\tau} - \theta\gamma\left(\overline{\tau} - \mu\right) > \gamma\mu, \tag{17}$$

which is equivalent to $\theta\gamma < \frac{\overline{\tau}-\gamma\mu}{\overline{\tau}-\mu}$. Therefore a solution exists, provided this condition holds.

To prove the uniqueness of the equilibrium it is sufficient to show that the left-hand side of (16) increases with $\tilde{\tau}_l$:

$$1 - \theta\gamma\frac{\partial E[\tau \mid \tau \geq \tilde{\tau}_l]}{\partial\tilde{\tau}_l} > 1 - \overline{\tau}\frac{\partial E[\tau \mid \tau \geq \tilde{\tau}_l]}{\partial\tilde{\tau}_l} > 0, \tag{18}$$

due to Assumption T2. Thus, for $\theta < \min\left\{\frac{\overline{\tau}-\gamma\mu}{\gamma(\overline{\tau}-\mu)}, \overline{\theta}\right\}$ there is a unique equilibrium $\tilde{\tau}_l$.[4]

Only if. If $\frac{\overline{\tau}-\gamma\mu}{\gamma(\overline{\tau}-\mu)} < \overline{\theta}$ and $\theta > \frac{\overline{\tau}-\gamma\mu}{\gamma(\overline{\tau}-\mu)}$, equation (16) has no solution. ■

The intuition for this result is as follows. The left-hand side of the inequality in (17) is the profit that firm l obtains if it retains the best possible manager, at the lowest possible salary that prevents this manager from being poached, which is $\theta\gamma\left(\overline{\tau} - \mu\right)$. The right-hand side of (17) is the profit that firm l obtains if it replaces its incumbent manager with a young manager. Consider some $\tau' = \overline{\tau} - \varepsilon$, where $\varepsilon > 0$ is sufficiently small. By continuity and T2, condition $\theta \leq \min\left\{\theta^*, \overline{\theta}\right\}$ implies that a type-l firm can choose to retain all managers with talent greater than $\tau' < \overline{\tau}$ and pay them $\theta\gamma\left(E[\tau \mid \tau \geq \tau'] - \mu\right)$, which would prevent poaching. A type-l firm would then consider decreasing its threshold further, until it reaches a value $\tilde{\tau}_l$, at which the threshold cannot be reduced further without inducing poaching. Thus, a unique no-poaching equilibrium must exist.

3.4. Equilibrium Properties

We cannot obtain a simple analytical solution unless we impose a specific functional form on F. However, without imposing any further structure, we can still prove a number of interesting properties of the no-poaching equilibrium.

The key implications of the no-poaching equilibrium for compensation and turnover are summarized in the next proposition.

Proposition 2 ***(Turnover and Compensation)*** *In a no-poaching equilibrium:*

1. *The retention threshold is higher in type-l firms:* $\tilde{\tau}_l > \tilde{\tau}_h$*;*
2. *The turnover rate is higher in type-l firms:* $\beta_l > \beta_h$*;*
3. *Incumbent manager compensation is higher in type-l firms:* $w_l > w_h$*.*

[4] Uniqueness here is in a generic sense. The equilibrium is not unique if parameters are such that $\theta = \theta^*$.

Proof. *1.* This follows from equations (15) and (16) and the fact that the left-hand side of (16) is increasing in $\tilde{\tau}_l$.

2. This follows from $\tilde{\tau}_l > \tilde{\tau}_h$ and the stationarity condition $\beta_i = \frac{1}{2-F(\tilde{\tau}_i)}$, $i = l, h$.

3. $w(\tilde{\tau}_i) = \theta\gamma(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu)$ is increasing in $\tilde{\tau}_i$, and $\tilde{\tau}_l > \tilde{\tau}_h$. ■

Proposition 2 holds because a less profitable firm finds it harder to retain its manager than does a more profitable firm. Thus, low-profitability firms need to offer their managers a larger share of their profits (i.e. a higher salary) to prevent their managers from being poached by high-profitability firms.

Proposition 3 ***(The profitability differential)*** *In type-l firms, the turnover rate β_l increases with θ, while in type-h firms turnover is independent of θ. Compensation for retained managers w_i increases with the profitability differential in both types of firms.*

Proof. It follows immediately from the inspection of conditions (12), (15) and (16). ■

As firms become more heterogeneous (θ increases), low-profitability firms find it harder to compete with high-profitability firms in the market for executives. To sustain a no-poaching equilibrium, low-profitability firms need to offer higher salaries to retain their managers. This in turn forces them to increase turnover by firing more managers.

Proposition 4 ***(Firm-specific skills)*** *In both types of firms, both compensation w_i and turnover β_i (weakly) increase with γ.*

Proof. It follows immediately from the inspection of conditions (12), (15) and (16). ■

A larger γ represents a situation in which firm-specific skills become less important. Turnover increases for two reasons. First, turnover increases because the value of a newly-hired manager is larger when firm-specific skills are less important. This effect is present even if firms face no competition for managers. When there is competition in the executive labor market, changes in the importance of firm-specific skills have an additional, indirect effect on turnover: an increase in γ makes poaching more attractive, which in turn requires firms to pay larger salaries to their retained managers to prevent poaching. As discussed above, higher salaries imply higher turnover because firms can no longer afford to keep mediocre managers.

4. Poaching Equilibrium

In this section, we define, characterize, and prove the existence of the second of the two possible types of equilibrium in our model: an equilibrium with managerial mobility across firms. We call this equilibrium a *poaching equilibrium.*

Here we have two qualitatively distinct possibilities. The first case happens when the number of poachers is larger than the number of qualified poaching targets. In this case, we say that qualified old managers are in short supply. We expect managers to benefit significantly from the existence of equilibrium poaching in this case. The second case happens when managers are in excess supply, in which case we expect type-h firms to benefit substantially from their ability to poach.

There is no theoretical reason to choose one case over the other. Thus, we characterize both cases, and discuss their somewhat different empirical implications. Here we establish the necessary conditions for each case to arise.

Case 1: Old managers in short supply. In this case, all incumbent managers from type-l firms are poached in equilibrium.[5] Let p denote the fraction of the h-firms that become poachers in equilibrium and, as before, let α_h denote the fraction of h-firms that retain their managers and β_h denote the fraction of h-firms that hire young managers. At the end of periods T and $T+1$, respectively, we have

	T	$T+1$
poachers	p	p
retained	α_h	$\beta_h(1-F(\tilde{\tau}_h))$
new hires	β_h	$\beta_h F(\tilde{\tau}_h)+\alpha_h$

The stationarity conditions write:

$$\begin{cases} \alpha_h = \beta_h(1-F(\tilde{\tau}_h)) \\ \beta_h = \alpha_h + \beta_h F(\tilde{\tau}_h) \\ \alpha_h + \beta_h + p = 1 \\ pH = (1-F(\tilde{\tau}_l))L \end{cases}. \tag{19}$$

The equilibrium fraction of new hires is $\beta_h = \left[1-\frac{(1-F(\tilde{\tau}_l))L}{H}\right]/(2-F(\tilde{\tau}_h))$. This fraction is positive (i.e. there are some junior hires in equilibrium) only if $H>(1-F(\tilde{\tau}_l))L$, which is a necessary condition for this case to occur.

[5]We caution against a literal interpretation of this case. Empirically, we are thinking about a situation in which there are relatively few qualified CEOs from small firms that could make the jump to a larger firm. That is, there could be many small firms, but only a few are breeding grounds for top CEOs. We could easily modify our model to fit this interpretation by, for example, assuming that only a fraction of the existing managers are viable poaching targets.

Case 2: Old managers in excess supply. In this case, no incumbent manager is retained by h-firms; the turnover rate is 100%.[6] The fractions of retained managers α_l and young hires β_l in l-firms are:

	T	$T+1$
retained	α_l	$\beta_l(1-F(\tilde{\tau}_l))\left[1-\frac{H}{\beta(1-F(\tilde{\tau}_l))L}\right]$
new hires	β_l	$\beta_l F(\tilde{\tau}_l)+\alpha_l+\frac{H}{L}$

The stationarity conditions write:

$$\begin{cases} \alpha_l = \beta_l(1-F(\tilde{\tau}_l)) - \frac{H}{L} \\ \beta_l = \beta_l F(\tilde{\tau}_l) + \alpha_l + \frac{H}{L} \\ \alpha_l + \beta_l = 1 \end{cases}, \tag{20}$$

implying $\beta_l = \left(1+\frac{H}{L}\right)/\left(2-F(\tilde{\tau}_l)\right)$ and $\alpha_l = \left(1-F(\tilde{\tau}_l)-\frac{H}{L}\right)/\left(2-F(\tilde{\tau}_l)\right)$. This equilibrium may exist only if $H \leq L(1-F(\tilde{\tau}_l))$.

4.1. Definition

Definition 2 ***(Poaching Equilibrium)*** *A poaching (stationary) equilibrium is a (time-invariant) set of salaries, retention thresholds, and turnover rates* $\{(\tilde{\tau}_l^p, w_l^p, \beta_l^p), (\tilde{\tau}_h^p, w_h^p, \beta_h^p + p)\}$ *such that:*

- *A firm of type* $i \in \{l, h\}$ *tries to retain any incumbent manager with talent above* $\tilde{\tau}_i^p$ *and offers her a salary* w_i^p*;*
- *A firm of type* l *dismisses any incumbent manager with talent below* $\tilde{\tau}_l^p$*, in which case it hires a young manager, who accepts to work for a salary of zero;*
- *A firm of type* h *dismisses any incumbent manager with talent below* $\tilde{\tau}_h^p$*, in which case it either hires a young manager or chooses to become a poacher. A poacher makes an offer to a manager working for a type-*l *firm.*
- *A manager working for a type-*l *firm that is the target of a poaching attempt accepts the poacher's offer. If there is no poaching attempt, a manager with talent above* $\tilde{\tau}_l^p$ *stays with her employer.*

[6] Again, the interpretation should not be literal; this is simply a case in which turnover at large firms is very high.

- *The salary w_h^p is such that no firm expects to gain from offering a higher salary to a manager from a type-h firm;*

- *The stationarity conditions hold;*

- *All players know $\{(\tilde{\tau}_l^p, w_l^p, \beta_l^p), (\tilde{\tau}_h^p, w_h^p, \beta_h^p + p)\}$.*

4.2. Existence

We now describe the derivation of a poaching equilibrium.

Because l-firms do not fight poaching attempts by h-firms, their retention threshold is determined by the no poaching condition for l-firms:

$$\tilde{\tau}_l^p - \gamma E[\tau \mid \tau \geq \tilde{\tau}_l^p] = 0. \tag{21}$$

Notice that given T2, a unique solution to this condition always exists, and is the same for either case of poaching equilibrium. Note also that, for given γ, the retention threshold for an l-firm in a poaching equilibrium is identical to the threshold for an h-firm in a no-poaching equilibrium: $\tilde{\tau}_l^p = \tilde{\tau}_h$.

The equilibrium salary for retained managers at l-firms is thus:

$$w_l^p = \gamma \left(E[\tau \mid \tau \geq \tilde{\tau}_l^p] - \mu \right). \tag{22}$$

Given $\tilde{\tau}_l^p$, we can now check whether we are in case 1 or case 2.

Case 1: $H > (1 - F(\tilde{\tau}_l^p))L$.

Because poachers are in excess supply, some h-firms without incumbent managers will be forced to hire young managers. Competition will then drive up the salaries of poached managers until h-firms are indifferent between poaching an old manager or hiring a young one:

$$\theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_l^p] - \hat{w}_h = \theta\gamma\mu. \tag{23}$$

This equation pins down the equilibrium poaching salary:

$$\hat{w}_h = \theta\gamma \left(E[\tau \mid \tau \geq \tilde{\tau}_l^p] - \mu \right). \tag{24}$$

Finally, the remaining equilibrium quantities are determined by

$$\tilde{\tau}_h^p - \gamma E[\tau \quad | \quad \tau \geq \tilde{\tau}_h^p] = 0 \tag{25}$$

$$w_h^p = \theta\gamma \left(E[\tau \mid \tau \geq \tilde{\tau}_h^p] - \mu\right). \tag{26}$$

Notice that, because l-firms offer w_l^p to all "retained" managers and h-firms offer $\hat{w}_h = w_h^p > w_l^p$ to all retained managers (from either type of firm), l-firms are unable to retain any manager. Sequential rationality is the only reason for l-firms to offer positive salaries to managers: if, for whatever reason, an h-firm does not try to poach those incumbent managers who have talents above the threshold, then an l-firm would want to offer w_l^p to prevent their managers from being poached by another l-firm.

Case 2: $H \leq (1 - F(\tilde{\tau}_l^p))L$.

In this case, all h-firms are poachers. We just need to determine $\hat{w}_h$. Suppose that a type-h poacher identifies a target j among the type-l firms that keep their manager and makes an offer

$$\hat{w}_h = w_l^p + \varepsilon, \tag{27}$$

where $\varepsilon > 0$ is arbitrarily small. By construction, firm j can never win a fight against a poacher that offers contract (27). Thus, because of Assumption C2, the target does not fight a poaching attempt, and the poacher always wins. A rational poaching attempt requires ε to be as small as possible, thus we let $\varepsilon \to 0$.

We need to check whether it is rational for an h-firm to become a poacher rather than hire a young manager:

$$\theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_l^p] - w_l^p = (\theta - 1)\,\gamma E[\tau \mid \tau \geq \tilde{\tau}_l^p] + \gamma\mu > \gamma\mu, \tag{28}$$

which always holds. Thus, assuming that a poaching equilibrium is possible, we have shown that, for each case there is only one set of equilibrium thresholds and salaries that characterize the equilibrium. This means that we have proven part of the following proposition:

Proposition 5 ***(Existence of poaching equilibrium)*** *A poaching equilibrium exists and is unique if and only if*

$$\theta \geq \theta^* = \frac{\overline{\tau} - \gamma\mu}{\gamma(\overline{\tau} - \mu)}. \tag{29}$$

Proof. *Only if.* The maximum that an l-firm is willing to pay in order to retain $\overline{\tau}$ is:

$$\overline{\tau} - w_l^{max}(\overline{\tau}) = \gamma\mu \Rightarrow w_l^{max}(\overline{\tau}) = \overline{\tau} - \gamma\mu. \tag{30}$$

The maximum that an h-firm is willing to pay in order to poach a manager with talent $\overline{\tau}$ is:

$$\gamma\theta\overline{\tau} - \hat{w}_h(\overline{\tau}) = \gamma\theta\mu \Rightarrow \hat{w}_h(\overline{\tau}) = \gamma\theta\overline{\tau} - \gamma\theta\mu. \tag{31}$$

Poaching can only occur if an l-firm is not willing to match an offer made by an h-firm to the best possible manager, i.e. when $\hat{w}_h(\overline{\tau}) \geq w_l^{max}(\overline{\tau})$. Hence a necessary condition for the existence of a poaching equilibrium is:

$$\theta \geq \frac{\overline{\tau} - \gamma\mu}{\gamma(\overline{\tau} - \mu)}. \tag{32}$$

If. Assuming that an equilibrium exists, we characterized one set of quantities for each possible (mutually exclusive) case. Condition (29) implies that an l-firm does not want to deviate and fight a poaching attempt because, if it cannot retain the best possible manager, it cannot win a fight for any manager. ■

Notice that condition (29) is the complement of condition $\theta \leq \min\left\{\theta^*, \overline{\theta}\right\}$. Thus, when poaching becomes possible, the no-poaching equilibrium is no longer feasible, and vice versa.

4.3. Equilibrium Properties

The key implications of the poaching equilibrium for compensation and turnover are summarized in the next proposition.

Proposition 6 ***(Turnover and Compensation)*** *In a poaching equilibrium:*

1. The retention threshold is (weakly) higher in type-h firms: $\tilde{\tau}_h^p \geq \tilde{\tau}_l^p$*;*

2. The turnover rate is higher in type-l firms in case 1, and lower in type-l firms in case 2.

3. Incumbent manager compensation is (weakly) higher in type-h firms: $w_h \geq w_l$.

Proof. Case 1. *1.* $\tilde{\tau}_h^p = \tilde{\tau}_l^p$ (immediate from inspection of (21) and (25)).

2. Turnover in l-firms is 100%.

3. Offered salary is $w_l^p < w_h^p$ (see (22) and (26)) and effective salary paid is zero in l-firms.

Case 2. *1.* $\tilde{\tau}_h^p = 1 > \tilde{\tau}_l^p$, because all h-firms are poachers.

2. Turnover in h-firms is 100%.

3. Salary is $\hat{w}_h = w_h^p$ for all old managers. ■

Proposition 7 (The profitability differential) *The profitability differential has no impact on the equilibrium choices of type-l firms. In case 1, an increase in* θ *increases the* *compensation offered to poached managers $\hat{w}_h$ and the wage offered to retained managers w_h^p. In case 2, θ has no impact compensation.*

Proof. Trivial. ■

In case 1, old managers in l-firms are in short supply, and thus are able to capture a sizeable part of the surplus created by an increase in θ, which is reflected in their increased compensation. In case 2, h-firms fully capture all of the surplus created by an increase in θ.

Proposition 8 ***(Firm-specific skills)*** *In both types of firms, the salaries w_i^p and $\hat{w}_h$ increase with γ.*

Proof. Trivial. ■

The intuition for the impact of γ on salaries is similar to that in the non-poaching equilibrium case.

5. Comparing the two equilibria

To facilitate the interpretation of the results, we now refer to the no-poaching equilibrium as the *old economy equilibrium*, and to the poaching equilibrium as the *new economy equilibrium*. Running the risk of abusing our freedom to interpret the model, we refer to type-l firms as small firms, and to type-h firms as large firms.

Old economy vs. New economy (Case 1).

We now compare the characteristics of the no-poaching equilibrium with those of the case-1 poaching equilibrium. For low values of γ and θ, turnover only occurs due to involuntary departures of less talented managers. As θ or γ increase, poaching becomes possible and thus voluntary departures of managers become possible. Hence, in the old economy we do not observe executive mobility across firms. Executives are appointed either internally or from a pool of outside managers. In contrast, in the new economy, the more profitable/larger firms poach talented managers from smaller, less profitable firms.

Remark 1 *Executive mobility jumps discontinuously from zero to $(1 - F(\tilde{\tau}_l^p))L$ once either θ or γ become large enough.*

In other words, the quality of the executive-firm match is improved once we move from the old economy to the new economy. Intuitively, as upward mobility becomes possible, large firms can free-ride on the information generated by the retention decisions of small firms and thus poach their best managers. Thus, large firms will attract the best managers from small firms. This poaching represents an improvement in the allocation of resources in the economy, because talent is complementary to size (i.e. productivity).

We now focus on comparing the behavior of large firms in each of the equilibria. Most of the available long-run evidence on executive turnover and compensation is from large firms only, thus we focus on these results because they can be directly compared with the existing evidence.

One advantage of our model is that we can compare the two different equilibria while keeping constant all the other characteristics of the economy. That is, by considering the case in which $\theta = \theta^*$, in which both equilibria are possible, we can determine what happens to compensation and turnover when moving form one equilibrium to another, while keeping everything else constant.

Remark 2 *The new economy has a higher level of executive turnover in large firms than the old economy does.*

Proof. Turnover in h-firms in the poaching equilibrium (case 1) is given by

$$\beta_h^p + p = \frac{1 + \frac{L}{H}}{2 - F(\tilde{\tau}_h^p)} > \frac{1}{2 - F(\tilde{\tau}_h)} = \beta_h. \tag{33}$$

■

In the new economy, large firms find executive turnover more desirable, because now they can replace their incumbent executives with the best executives from smaller firms. This in turn increases turnover in large firms, once the new economy regime kicks in.

Remark 3 *In large firms, executive compensation in the new economy is larger than executive compensation in the old economy.*

Proof. The average pay at h-firms in the new economy is given by

$$\left(\alpha_h^p + p\right) w_h^p = \frac{1 - F(\tilde{\tau}_h^p) + \frac{(1-F(\tilde{\tau}_l^p))L}{H}}{2 - F(\tilde{\tau}_h^p)} w_h^p, \tag{34}$$

and the average pay at h-firms in the old Economy is given by

$$\alpha_h w_h = \frac{1 - F(\tilde{\tau}_h)}{2 - F(\tilde{\tau}_h)} w_h. \tag{35}$$

The result then follows from $\tilde{\tau}_h^p = \tilde{\tau}_h$ and $w_h^p = w_h$. ■

We conclude that mobility, compensation, and turnover in large firms increase discontinuously when the new economy kicks in. The reason for this change is quite simple: now that poaching is feasible, large firms have more options and thus replace their managers more often. The competition for managers bids up the salaries of poached managers.

There are also a number of less straightforward results. For example, we can show that pay inequality across firms of different sizes is more pronounced in the new economy:

Remark 4 *The firm-size pay premium is larger in the new economy than in the old economy.*

This result follows immediately from the fact that the average pay in large firms increases as the new economy kicks in (Remark 3), while pay in small firms does not change. In fact, this result is even stronger in the more general case in which we do not impose assumption T2. In that case, for some parameters, the average pay in small firms also decreases as one moves to the new equilibrium, which increases the firm-size pay premium even further.

Remark 5 *In large firms, the positive relation between firm size and executive pay is stronger in the new economy than in the old economy.*

Proof. The pay-size sensitivity at h-firms in the old economy is given by

$$\frac{\partial \alpha_h w_h}{\partial \theta} = \frac{1 - F(\tilde{\tau}_h)}{2 - F(\tilde{\tau}_h)} \gamma \left(E[\tau \mid \tau \geq \tilde{\tau}_h] - \mu \right), \tag{36}$$

and in the new economy it is

$$\frac{\partial \alpha_h^p w_h^p}{\partial \theta} = \frac{1 - F(\tilde{\tau}_h^p) + \frac{(1 - F(\tilde{\tau}_l^p))L}{H}}{2 - F(\tilde{\tau}_h^p)} \gamma \left(E[\tau \mid \tau \geq \tilde{\tau}_h^p] - \mu \right). \tag{37}$$

The result then follows from $\tilde{\tau}_h^p = \tilde{\tau}_h$ and $w_h^p = w_h$. ■

This result fits the empirical evidence quite well. The pay-size relationship was relatively flat until the mid 1970s, when it became much steeper.

Old economy vs. New economy (Case 2).

In this case, both Remarks 1 and 2 remain valid. However, Remarks 3 to 5 no longer hold. Intuitively, when large firms are relatively more powerful than managers, the improvement in the quality of the matching that is characteristic of the new economy benefits the shareholders of large firms more than their managers. The results of Case 1 seem to fit the evidence better and also the popular perception that CEOs benefit more from improved profitability than shareholders.

6. Discussion and Extensions

We have so far developed a simplified version of our model. In Subsection 6.1, we first briefly compare this simple version with an even simpler case: a model in which learning is symmetric. In Subsection 6.2, we introduce effort and incentive pay. In Subsection 6.3, we characterize the equilibrium conditions for the general case of $\delta \geq 0$.

6.1. Symmetric Learning

Under symmetric information, the salary function is the same for everyone: $w(\tau)$. An h-type firm retains a manager of type τ if

$$\theta\tau - w(\tau) \geq \theta\gamma\mu, \tag{38}$$

and poaches a manager of type τ unless

$$\theta\gamma\tau - w(\tau) \leq \theta\gamma\mu. \tag{39}$$

Thus no-poaching salaries are given by

$$w(\tau) = \max\{\theta\gamma(\tau - \mu), 0\}. \tag{40}$$

An l-firm retains type τ if

$$\tau - \max\{\theta\gamma(\tau - \mu), 0\} \geq \gamma\mu. \tag{41}$$

Thus, firm l always retains all types $\tau \in [\gamma\mu, \mu]$ and pays them nothing (in all equilibria), and also retains $\tau > \mu$ only if

$$\tau - \theta\gamma(\tau - \mu) \geq \gamma\mu, \tag{42}$$

which holds for sure if $\gamma\theta < 1$, but may also hold for some $\gamma\theta > 1$, in which case the maximum τ that the firm retains is

$$\tau(\theta) = \frac{\gamma\mu(1-\theta)}{1-\theta\gamma}. \tag{43}$$

We have

$$\tau'(\theta) = \gamma\mu\frac{\gamma - 1}{(1-\theta\gamma)^2} < 0, \tag{44}$$

that is, the maximum type that is retained falls with θ.

An equilibrium in which all types are retained exists only if

$$\overline{\tau} - \theta\gamma\left(\overline{\tau} - \mu\right) \geq \gamma\mu, \tag{45}$$

which implies the following threshold for θ:

$$\theta \leq \frac{\overline{\tau} - \gamma\mu}{\gamma\left(\overline{\tau} - \mu\right)} = \theta^{*}. \tag{46}$$

We first note that the condition for the existence of a no-poaching equilibrium is exactly the same as that in the case of asymmetric learning. However, if $\theta > \theta^{*}$, the new equilibrium will involve some poaching, but only the most talented managers will be poached. An l-firm always retains all $\tau \in \left[\gamma\mu, \tau\left(\theta\right)\right]$, where $\tau\left(\theta\right) > \mu$ for any finite θ.

Thus, under symmetric information, there is no sharp discontinuity in executive mobility. As θ goes above θ^{*}, the fraction of poached managers grows monotonically with θ and is given by $\left[\tau\left(\theta\right), \overline{\tau}\right]$.

Let us summarize what our assumption of asymmetric learning is contributing to the analysis. First, from a purely methodological viewpoint, this assumption allows us to compare the two equilibria *while keeping all other parameters constant*. This is convenient because it allows us to analyze the differences in the characteristics of executive labor markets that are explained only by the nature of the equilibrium, and not by any of the other parameters in the economy. This is obviously not possible in the symmetric learning case, because the progression from one equilibrium to another is smooth.

Second, with asymmetric learning, drastic changes in the characteristics of executive labor markets may occur even when key parameters, such as firm heterogeneity (θ) and the importance of firm-specific skills (γ), change slowly and continuously. That is, in our model, asymmetric learning is a natural way of generating a discontinuity in an otherwise continuous environment.[7]

Third, the asymmetric learning case allows us to compare the relative efficiency of the two types of equilibria. Under symmetric information, turnover is always efficient. Firms replace their managers either when firms expect outside managers to be better than incumbents or when incumbents are poached. In the latter case, better managers move to more productive firms, which improves allocational efficiency. Asymmetric learning generates two types of

[7] We note however that asymmetric learning is not sufficient for such discontinuity. It is possible to make the transition smooth again, for example, by allowing for a continuum of firm types. We make only the weaker claim that asymmetric learning makes discontinuities more likely to occur.

turnover, one good and one bad. The good (efficient) turnover is the same as that in the symmetric information case. Bad turnover happens when firms fire some managers, who have talent above the average, to avoid lowering their pay and inviting poaching. Thus, the new economy equilibrium is more efficient than the old economy equilibrium, because in the new economy there is both less bad turnover and more good turnover (poaching).

Finally, we note that the asymmetric learning model has different empirical implications. For example, in the asymmetric learning case, we can meaningfully speak of a firm size-pay premium, in the sense that, controlling for all other observable executive characteristics, executive pay is a function of firm size. In the symmetric learning case, all executives with the same general talent are paid the same, regardless of which firms they work for.

6.2. Effort and Incentive Pay

Our model is silent about the composition of executive pay. In this subsection, we introduce a simple moral hazard problem that allows us to analyze the composition of pay and compare them across the two equilibria.

Let us now assume that managerial effort is another productive input. Effort is measured by $e \in (0,1)$, which is private information to the manager. As before, managers are risk-neutral, have no initial wealth, and are protected by limited liability. We assume that effort imposes a nonpecuniary cost to the manager according to a quadratic cost function $c(e) = \frac{e^2}{2}$.

We modify the technology to incorporate effort:

$$R_{jt}\left(i_j, s_{jt}, \tau_{jt}, e_{jt}\right) = \begin{cases} 1, & \text{with probability } i_j s_{jt} \tau_{jt} e_{jt} \\ 0 & \text{with probability } 1 - i_j s_{jt} \tau_{jt} e_{jt} \end{cases}. \tag{47}$$

We keep the same restrictions on the these variables as before.

Each firm now offers their managers a contract (w, b), where $w \geq 0$ is a fixed salary and $b \in [0,1]$ is the fraction of the output that is given to the manager. Given the binomial nature of the technology, we do not need to consider more complicated contracts.

We replace Assumption 1 with its equivalent for this case:

Assumption T1’ $\underline{\tau} \leq \gamma\sqrt{\sigma^2 + \mu^2}$.

Here σ^2 denotes the variance of F.

We also need to impose a different restriction on the derivative of $E[\tau^2 \mid \tau \geq \tilde{\tau}]$. We first define the following function:

$$\eta(\tilde{\tau}) \equiv \frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} \frac{\tilde{\tau}}{E[\tau^2 \mid \tau \geq \tilde{\tau}]}. \tag{48}$$

Notice that $\eta(\tilde{\tau})$ is the elasticity of the second non-centered moment of the truncated distribution $G(\tau \mid \tau \geq \tilde{\tau})$ with respect to the truncation point $\tilde{\tau}$. We now assume:

Assumption T2' $\eta(\tilde{\tau}) \leq 2$ for all $\tilde{\tau} \in [\underline{\tau}, \overline{\tau}]$.

Assumption T2' is not very restrictive. For example, we have that:

Lemma 2 *If f is log-concave, then $\eta(\tilde{\tau}) \leq 2$ holds for all $\tilde{\tau} \in [\underline{\tau}, \overline{\tau}]$.*

Proof. See the Appendix. ■

6.2.1. Monopoly

To fix ideas, we briefly develop here a benchmark case in which there is no competition.

Suppose initially that there is only one firm. The firm knows the talent of its incumbent manager, τ. Because there are no other firms, if the firm wishes to replace its manager, it must hire a young manager.

Let π_i denote the profit of a firm of type i. Suppose first that the firm decides to retain its manager. The program of a type-i firm is to:

$$\max_{b,w} E[\pi_i] = (1-b)i\tau e - w, \tag{49}$$

subject to

$$\begin{cases} bi\tau - e = 0 & (IC) \\ bi\tau e - \frac{e^2}{2} + w \geq 0 & (IR) \\ w \geq 0 & (LL) \end{cases} . \tag{50}$$

The first constraint is the incentive compatibility (IC) constraint for the manager, the second one is her participation or individual rationality (IR) constraint, and the last one is the limited liability (LL) constraint. In an optimal contract, LL binds and w is always zero. This is true in all the cases that we consider. Thus, to simplify the exposition we ignore w from now on. Notice that IC implies that IR is not binding. The optimal bonus is $b^M = \frac{1}{2}$, where the superscript M refers to the optimal solution in the monopolistic scenario.

This bonus is independent of the type of the firm. The (expected) profit from retaining the incumbent manager is given by $\pi_i^o = \frac{i^2\tau^2}{4}$.

Suppose now that the firm decides to fire its manager and replace her with a young manager. Using the fact that $w = 0$, the program of a type-i firm is now:

$$\max_b E[\pi_i] = E\left[(1-b)i\gamma\tau e\right] \tag{51}$$

subject to

$$\begin{cases} b\gamma i\tau - e = 0 & (IC) \\ b\gamma i\tau e - \frac{e^2}{2} \geq 0 & (IR) \end{cases}. \tag{52}$$

The optimal bonus is still $b^M = \frac{1}{2}$. The expected profit from hiring a young manager

$$\pi_i^y = \frac{i^2\gamma^2 E_F[\tau^2]}{4} = \frac{i^2\gamma^2\left(\sigma^2+\mu^2\right)}{4}. \tag{53}$$

The firm retains its incumbent manager if and only if $\pi_i^o \geq \pi_i^y$, or

$$\tau \geq \tilde{\tau}^M \equiv \gamma\sqrt{\sigma^2+\mu^2}. \tag{54}$$

6.2.2. Existence of a No-Poaching Equilibrium

We now describe the derivation of a no-poaching equilibrium.

If a firm chooses to hire a young manger, it faces no competition. Thus, the optimal bonus is the same as that in the monopolistic case: $b_i^y = \frac{1}{2}$, $i = l, h$. Thus, in what follows we only need to derive the equilibrium values for $(b_i, \tilde{\tau}_i)$, $i = l, h$.

A firm of type i that retains a manager with talent τ_i and pays a bonus b_i has an expected profit of $b_i(1-b_i)i^2\tau_i^2$. Under no poaching, firm i retains the manager if

$$b_i(1-b_i)i^2\tau_i^2 \geq \frac{i^2\gamma^2 E[\tau^2]}{4}. \tag{55}$$

This rule implies a threshold decision (given b_i) to retain the manager only if $\tau_i \geq \tilde{\tau}_i$, where

$$\tilde{\tau}_i = \sqrt{\frac{\gamma^2 E[\tau^2]}{b_i\left(1-b_i\right)4}},\ i = l, h. \tag{56}$$

This condition links the equilibrium thresholds to the offers that are made to the incumbent managers. Note that $\tilde{\tau}_i$ is an increasing function of $b_i \in \left[\frac{1}{2}, 1\right]$.

We now derive the no-poaching condition for an h-firm. The minimum offer that a manager working for firm i would accept is $\frac{b_i}{i\gamma}$. If $\theta\gamma > 1$, a manager from a type-l firm would accept to work for a type-h firm even if the bonus was lower than b_l. Thus the minimum offer that a type-h firm makes to a type-i incumbent manager is $b_i' = \max\left\{\frac{b_i}{i\gamma}, \frac{1}{2}\right\}$, because it is never optimal to offer a bonus that is lower than $\frac{1}{2}$ (the optimal bonus in the monopolistic case).

Assuming that the minimum offer is made, if the poaching attempt is successful, firm h's profit is

$$b_i' \left[1 - b_i'\right] \theta^2 \gamma^2 E[\tau^2 \mid \tau \geq \tilde{\tau}_i]. \tag{57}$$

This profit is decreasing in $b_i' \in \left[\frac{1}{2}, 1\right]$.

In a no-poaching equilibrium, this profit must satisfy two conditions. First, $b_i' = \frac{b_i}{i\gamma}$, otherwise a type-$h$ firm could offer a bonus of $\frac{1}{2}$ to poach a manager from a type-l firm. Second, this profit must not be larger than the one obtained by hiring a young manager. These two conditions imply the following one:

$$\frac{b_i}{i\gamma}\left[1 - \frac{b_i}{i\gamma}\right] E[\tau^2 \mid \tau \geq \tilde{\tau}_i] \leq \frac{E[\tau^2]}{4}. \tag{58}$$

Unless $b_i = \frac{1}{2}$, the condition above cannot hold with slack in equilibrium, because if it did, firm i could reduce b_i and still avoid the poaching of its manager. Thus, in a no-poaching equilibrium, either condition (11) must hold with equality or $b_i = \frac{1}{2}$. We have that:

$$b_l = \max\left\{\frac{\theta\gamma}{2}\left(1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 \mid \tau \geq \tilde{\tau}_l]}}\right), \frac{1}{2}\right\}, \text{and} \tag{59}$$

$$b_h = \max\left\{\frac{\gamma}{2}\left(1 + \sqrt{1 - \frac{E[\tau^2]}{E[\tau^2 \mid \tau \geq \tilde{\tau}_h]}}\right), \frac{1}{2}\right\}, \tag{60}$$

which, together with condition (56) determine the equilibrium pairs $(b_l, \tilde{\tau}_l)$ and $(b_h, \tilde{\tau}_h)$, if an equilibrium exists.

The following proposition establishes the necessary and sufficient condition for the existence of a unique no-poaching equilibrium.

Proposition 9 ***(Existence of no-poaching equilibrium)*** *A no-poaching equilibrium ex-*

ists and is unique if and only if $\theta \leq \min\left\{\theta^{**}, \overline{\theta}\right\}$, *where*

$$\theta^{**} \equiv \frac{\tau^{*} + \sqrt{\tau^{*2} - \gamma^{2} E[\tau^{2}]}}{\gamma\left(\tau^{*} + \sqrt{\tau^{*2} - E[\tau^{2} \mid \tau \geq \tau^{*}]}\right)}, \tag{61}$$

and

$$\tau^{*} = \arg\max_{\tilde{\tau}_l \in \left[\underline{t}, \overline{t}\right]} (1 - b_l) b_l \tilde{\tau}_l^{2}. \tag{62}$$

Proof. See the Appendix. ■

6.2.3. Existence of a Poaching Equilibrium

We now describe the derivation of a poaching equilibrium. As before, $b_i^y = \frac{1}{2}$, $i = l, h$.

For brevity, we consider only Case 1. If all managers from l- firms are poached, h-firms compete for managers, driving the poaching bonus to

$$\hat{b}_h = \frac{1}{2}\left(1 + \sqrt{1 - \frac{E[\tau^{2}]}{E[\tau^{2} \mid \tau \geq \tilde{\tau}_l]}}\right). \tag{63}$$

Because both h and l firms only avoid poaching by firms of their own type, we have $\tilde{\tau}_l^p = \tilde{\tau}_h^p = \tilde{\tau}^p$ and

$$b_l^p = b_h^p = \max\left\{\frac{\gamma}{2}\left(1 + \sqrt{1 - \frac{E[\tau^{2}]}{E[\tau^{2} \mid \tau \geq \tilde{\tau}^{p}]}}\right), \frac{1}{2}\right\}. \tag{64}$$

For a no poaching equilibrium to exist, we need an l-firm to be unwilling to fight for a manager of type τ^{*}, which implies the following proposition

Proposition 10 ***(Existence of poaching equilibrium)*** *A poaching equilibrium exists and is unique if and only if* $\theta \geq \theta^{**}$.

Proof. It follows from the same arguments as in the proof of Proposition 9. ■

6.2.4. Comparing the Two Equilibria

Most of the results proved in the case of no effort also hold (with the appropriate modifications) in the current case. For brevity, here we only focus on the new results that are related to b, which is our measure of pay-performance sensitivity. Again, here we consider the case of $\theta = \theta^{**}$.

Remark 6 *In large firms, the pay-performance sensitivity in the new economy is larger than the pay-performance sensitivity in the old economy.*

Proof. It follows from the fact that $\hat{b}_h > b_h^p = b_h$. ■

This result follows from the fact that more intense competition for managers in the poaching equilibrium bids up the pay for the poached managers. Limited liability then implies that total pay and performance-based pay are positively linked. This result also implies that, in the new economy, the pay-performance sensitivity is larger for experienced managers hired from the outside than for experienced inside managers. Intuitively, because outside managers lack firm-specific skills, their performance-based bonuses must be larger so that outside managers can expect the same pay as managers retained by large firms.

6.3. The General Case

We finally focus on the case in which $\delta \geq 0$. In this case, the dynamics of the model are at play, and firms also compare the costs of hiring a unknown young manager today with the benefits of retaining this manager in the next period. Learning about τ gives the firm an option to retain exactly those managers who are revealed to have high talent.

The analysis in this section reveals the robustness of the previous conclusions in a fully dynamic setup. Furthermore, it gives us some additional insights on the role of learning about talent. Because the analysis here is also more complex, we revert to the initial case in which there is no effort, although the analysis of the case with effort is not qualitatively different.

We start with a no-poaching equilibrium. As we focus on stationary equilibria, we omit the time subscripts.

Let $\pi_i^o(\tau)$ denote the one-period profit of a firm of type i that keeps an old manager with talent τ and let π_i^y denote the one-period profit of a firm of type i that hires a young manager. The value function if the firm retains an old manager is

$$V_i^o(\tau) \equiv \pi_i^o(\tau) + \delta V_i^y, \tag{65}$$

and the value function if it hires a young manager is

$$V_i^y \equiv \pi_i^y + \delta F(\tilde{\tau}_i) V_i^y + \delta(1 - F(\tilde{\tau}_i)) E[V_i^o(\tau) \mid \tau \geq \tilde{\tau}_i], \tag{66}$$

which implies that a manager is retained if

$$V_i^o(\tau) - V_i^y = \pi_i^o(\tau) - \pi_i^y - \delta\left(1 - F(\tilde{\tau}_i)\right)\left(E\left[V_i^o(\tau) \mid \tau \geq \tilde{\tau}_i\right] - V_i^y\right) \geq 0. \tag{67}$$

Stationarity implies that

$$E\left[V_i^o(\tau) \mid \tau \geq \tilde{\tau}_i\right] - V_i^y = \frac{\delta\left(1 - F(\tilde{\tau}_i)\right)}{1 + \delta(1 - F(\tilde{\tau}_i))}\left(E\left[\pi_i^o \mid \tau \geq \tilde{\tau}_i\right] - \pi_i^y\right) \equiv K_i(\tilde{\tau}_i). \tag{68}$$

Notice that $K_i(\tilde{\tau}_i)$ is the equilibrium option value of hiring a young manager.

We can then simplify the condition that determines the retention of an old manager to

$$\pi_i^o(\tau) \geq \pi_i^y + K_i(\tilde{\tau}_i). \tag{69}$$

In a no-poaching equilibrium, the offer made by a potential h poacher to an i-firm is such that the poacher is indifferent between poaching an old manager or hiring a young manager:

$$\pi_h^p(\tilde{\tau}_i) = \pi_h^y + K_h(\tilde{\tau}_h). \tag{70}$$

This condition can be rewritten as follows:

$$\gamma\theta E[\tau \mid \tau \geq \tilde{\tau}_i] - \hat{w}_h(\tilde{\tau}_i) = \gamma\theta\mu + K_h(\tilde{\tau}_h), \tag{71}$$

where $\hat{w}_h(\tilde{\tau}_i)$ is the salary that a potential h poacher is willing to pay for a manager with ability above the threshold $\tilde{\tau}_i$. We can similarly define the salary $\hat{w}_l(\tilde{\tau}_i)$ that a potential l poacher is willing to pay for a manager with ability above the threshold $\tilde{\tau}_i$ from the following condition:

$$\gamma E[\tau \mid \tau \geq \tilde{\tau}_i] - \hat{w}_l(\tilde{\tau}_i) = \gamma\mu + K_l(\tilde{\tau}_l). \tag{72}$$

We can see that an h-firm has a higher immediate gain from hiring an old manager. However, the option value of an h-firm may be higher than the option value of an l-firm. Therefore, we do not know a priori which of $\hat{w}_h(\tilde{\tau}_i)$ and $\hat{w}_l(\tilde{\tau}_i)$ is larger.

The offer made by a firm of type i in order to retain managers with ability $\tau \geq \tilde{\tau}_i$ is given by

$$w_i(\tilde{\tau}_i) = \max\{\hat{w}_l(\tilde{\tau}_i), \hat{w}_h(\tilde{\tau}_i)\}. \tag{73}$$

The thresholds $\tilde{\tau}_i$ are given by the following equations:

$$\theta\tilde{\tau}_h - w_h(\tilde{\tau}_h) = \gamma\theta\mu + K_h(\tilde{\tau}_h), \tag{74}$$

$$\tilde{\tau}_l - w_l(\tilde{\tau}_l) = \gamma\mu + K_l(\tilde{\tau}_l). \tag{75}$$

For brevity, here we consider only the case in which $\hat{w}_h(\tilde{\tau}) \geq \hat{w}_l(\tilde{\tau})$. The analysis of this case is analogous to the case with $\delta = 0$. This case always holds if, for example, δ is not too large. We note though that the case in which $\hat{w}_h(\tilde{\tau}) < \hat{w}_l(\tilde{\tau})$ creates no difficulties. This case delivers the same qualitative implications at the transition threshold and is omitted here just for brevity of exposition.

Under the assumption that $\hat{w}_h(\tilde{\tau}) \geq \hat{w}_l(\tilde{\tau})$, we obtain the retention threshold in h-firms:

$$\tilde{\tau}_h = \gamma E[\tau \mid \tau \geq \tilde{\tau}_h]. \tag{76}$$

The option value does not affect the retention threshold for an h-firm. The intuition for this result is that the firm can recover all of the surplus created by the option of hiring a young manager by decreasing the reward offered to an old manager.

Salaries for retained managers become

$$w_i(\tilde{\tau}_i) = \max\left\{\gamma\theta\left(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu\right) - K_h(\tilde{\tau}_h), 0\right\}. \tag{77}$$

The retention threshold for an l-firm is given by:

$$\tilde{\tau}_l - \max\left\{\gamma\theta\left(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu\right) - K_h(\tilde{\tau}_h), 0\right\} - \gamma\mu - K_l(\tilde{\tau}_l) = 0. \tag{78}$$

Proposition 11 ***(Existence of no-poaching equilibrium)*** *A no-poaching equilibrium exists and is unique if and only if* $\theta \leq \min\left\{\theta^{***}, \overline{\theta}\right\}$, *where*

$$\theta^{***} \equiv \frac{\overline{\tau} - \gamma\mu}{\gamma(\overline{\tau} - \mu) - \delta(1-\gamma)\int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau)}, \tag{79}$$

Proof. See the Appendix. ■

Using the same arguments as in Section 4, we can easily show that the threshold for the existence of a unique poaching equilibrium is also $\theta \geq \theta^{***}$. We thus omit this proof for brevity. Similarly, it is possible to prove all previous results; the equilibrium values are different, but all results are qualitatively the same. Thus, we conclude that none of the results derived in Sections 2 to 4 depend on the assumption of $\delta = 0$.

All new results in this section come from the new threshold θ^{***}. We first note that, as $\delta \to 0$, $\delta(1-\gamma)\int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau) \to 0$ and thus $\theta^{***} \to \theta^*$. We also have that $\theta^{***} \geq \theta^*$, which implies that the no-poaching equilibrium can be sustained for larger values of the productivity differential θ. Intuitively, as the option to retain an old manager becomes less valuable, the relative value of poaching a manager increases, which makes poaching more likely to occur in equilibrium.

The value to an h-firm of hiring a young manager is increasing in $K_h(\tilde{\tau}_h)$, which is equal to $\theta\delta(1-\gamma)\int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau)$. The discount factor δ is positively related to $K_h(\tilde{\tau}_h)$. An alternative interpretation of δ is that it represents the probability that the firm will be in business in one period from now. Thus, tougher competition in product markets may lead a decrease in δ. Our results show that increasing product market competition could be yet another reason for an increase in executive labor market mobility.

7. Relation to the Literature

To be done.

8. Conclusions

To be done.

9. Appendix

9.1. Proof of Lemma 1

$$\begin{aligned}
\frac{\partial E[\tau \mid \tau \geq \tilde{\tau}]}{\partial \tilde{\tau}} &= \frac{f(\tilde{\tau})}{1-F(\tilde{\tau})}\{E[\tau \mid \tau \geq \tilde{\tau}] - \tilde{\tau}\} && (80)\\
&= \frac{f(\tilde{\tau})}{[1-F(\tilde{\tau})]^2}\left\{\int_{\tilde{\tau}}^{\bar{t}} \tau f(\tau)\, d\tau - \tilde{\tau}[1-F(\tilde{\tau})]\right\} && (81)\\
&= \frac{f(\tilde{\tau})}{[1-F(\tilde{\tau})]^2}\left\{\int_{\tilde{\tau}}^{\bar{t}} (\tau - \tilde{\tau}) f(\tau)\, d\tau\right\} && (82)
\end{aligned}$$

Integrating by parts yields

$$
\begin{aligned}
\frac{f(\tilde{\tau})}{[1-F(\tilde{\tau})]^2}\left\{(\tau-\tilde{\tau})\,F(\tau)]_{\tilde{\tau}}^{\bar{t}}-\int_{\tilde{\tau}}^{\bar{t}}F(\tau)\,d\tau\right\} &= \\
&= \frac{f(\tilde{\tau})}{[1-F(\tilde{\tau})]^2}\left\{\bar{t}F(\bar{t})-\tilde{\tau}F(\bar{t})-\int_{\tilde{\tau}}^{\bar{t}}F(\tau)\,d\tau\right\} \\
&= \frac{f(\tilde{\tau})\int_{\tilde{\tau}}^{\bar{t}}[1-F(\tau)]\,d\tau}{[1-F(\tilde{\tau})]^2}
\end{aligned}
$$

Now we take the limit as $\tilde{\tau} \to \overline{\tau}$

$$\lim_{\tilde{\tau}\to\overline{\tau}}\frac{dE[\tau \mid \tau \geq \tilde{\tau}]}{d\tilde{\tau}} = \lim_{\tilde{\tau}\to\overline{\tau}}\frac{f(\tilde{\tau})\int_{\tilde{\tau}}^{\bar{t}}[1-F(\tau)]\,d\tau}{[1-F(\tilde{\tau})]^2} \tag{83}$$

Using L'Hopital's rule twice,

$$
\begin{aligned}
& \lim_{\tilde{\tau}\to\overline{\tau}}\frac{f'(\tilde{\tau})\int_{\tilde{\tau}}^{\bar{t}}[1-F(\tau)]\,d\tau-[1-F(\tilde{\tau})]\,f(\tilde{\tau})}{-2\,[1-F(\tilde{\tau})]\,f(\tilde{\tau})} && (84)\\
= & \lim_{\tilde{\tau}\to\overline{\tau}}\frac{f'(\tilde{\tau})\int_{\tilde{\tau}}^{\bar{t}}[1-F(\tau)]\,d\tau}{-2\,[1-F(\tilde{\tau})]\,f(\tilde{\tau})}+\frac{1}{2} && (85)\\
= & \lim_{\tilde{\tau}\to\overline{\tau}}\frac{f''(\tilde{\tau})\int_{\tilde{\tau}}^{\bar{t}}[1-F(\tau)]\,d\tau-[1-F(\tilde{\tau})]\,f'(\tilde{\tau})}{2\,[f(\tilde{\tau})]^2-2\,[1-F(\tilde{\tau})]\,f'(\tilde{\tau})}+\frac{1}{2}=\frac{1}{2} && (86)
\end{aligned}
$$

which implies (by continuity) that $\frac{\partial E[\tau|\tau\geq\tilde{\tau}]}{\partial\tilde{\tau}} \leq \overline{\tau}$ in a neighborhood of $\overline{\tau}$.

9.2. Proof of Lemma 2

If f is a log-concave density, then $V'[\tau \mid \tau \geq \tilde{\tau}] \leq 0$ and $E'[\tau \mid \tau \geq \tilde{\tau}] \leq 1$ (see e.g. An (1988)). We have

$$\frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial\tilde{\tau}} - 2E[\tau \mid \tau \geq \tilde{\tau}]\frac{\partial E[\tau \mid \tau \geq \tilde{\tau}]}{\partial\tilde{\tau}} \leq 0, \tag{87}$$

therefore

$$\frac{\partial E[\tau^2 \mid \tau \geq \tilde{\tau}]}{\partial\tilde{\tau}} \leq 2E[\tau \mid \tau \geq \tilde{\tau}]. \tag{88}$$

Replacing in the elasticity we need the following condition to hold:

$$\frac{2E[\tau \mid \tau \geq \tilde{\tau}]\tilde{\tau}}{E[\tau^2 \mid \tau \geq \tilde{\tau}]} \leq 2 \tag{89}$$

which can be rewritten as:

$$\tilde{\tau}\int_{\tilde{\tau}}^{\overline{\tau}} \frac{\tau f(\tau)}{1-F(\tilde{\tau})} \leq \int_{\tilde{\tau}}^{\overline{\tau}} \frac{\tau^2 f(\tau)}{1-F(\tilde{\tau})}, \tag{90}$$

which always holds.

9.3. Proof of Proposition 9

Existence: The equilibrium values $\tilde{\tau}_i$ and b_i for a type-l firm are given by:

$$\begin{cases} b_l = \max\left\{\frac{1}{2}, \frac{\theta\gamma}{2}\left(1+\sqrt{1-\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tilde{\tau}_l]}}\right)\right\} \\ (1-b_l)b_l\tilde{\tau}_l^2 = \frac{\gamma^2 E[\tau^2]}{4} \end{cases} \tag{91}$$

Under Assumption T1', for $\tilde{\tau}_l = \underline{\tau}$, $(1-b_l)b_l\underline{\tau}_l^2 \leq \frac{\gamma^2 E[\tau^2]}{4}$.

Let

$$\tau^* = \arg\max(1-b_l)b_l\tilde{\tau}_l^2, \tag{92}$$

which for now we assume is uniquely defined. For $\tilde{\tau}_l = \tau^*$:

$$(1-b_l)b_l\tau^{*2} \geq \frac{\gamma^2 E[\tau^2]}{4} \text{ iff } b_l \leq \frac{1}{2}\left(1+\sqrt{1-\frac{\gamma^2 E[\tau^2]}{\tau^{*2}}}\right). \tag{93}$$

The maximum offer that an h-firm would make to a manager given $\tilde{\tau}_l = \tau$ is:

$$b_l' = \frac{1}{2}\left(1+\sqrt{1-\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tau^*]}}\right). \tag{94}$$

Therefore if $b_l'\theta\gamma \leq \frac{1}{2}\left(1+\sqrt{1-\frac{\gamma^2 E[\tau^2]}{\tau^{*2}}}\right)$, then $(1-b_l)b_l\tau^{*2} \geq \frac{\gamma^2 E[\tau^2]}{4}$. The former always holds if:

$$\frac{\theta\gamma}{2}\left(1+\sqrt{1-\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tau^*]}}\right) \leq \frac{1}{2}\left(1+\sqrt{1-\frac{\gamma^2 E[\tau^2]}{\tau^{*2}}}\right) \tag{95}$$

If the condition given by equation (95) holds, then an equilibrium exists.

Uniqueness We only need to show that τ^* is uniquely defined. For $b_l = \frac{1}{2}$, the function $(1-b_l)b_l\tilde{\tau}_l^2$ is strictly increasing and the proof is trivial.

For $b_l = \frac{\theta\gamma}{2}\left(1+\sqrt{1-\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tilde{\tau}_l]}}\right)$, let $(1-b_l)b_l\tilde{\tau}_l^2 \equiv B$, then

$$\begin{aligned}\frac{\partial B}{\partial \tilde{\tau}_l} &= 2\tilde{\tau}_l b_l' \gamma\theta(1-b_l'\gamma\theta) - \tilde{\tau}_l^2\gamma\theta(2b_l'\gamma\theta-1)\frac{\partial b_l'}{\partial \tilde{\tau}_l} \\ &= 2\tilde{\tau}_l\gamma\theta\Big(b_l'(1-b_l'\theta\gamma) - \frac{\tilde{\tau}_l(2b_l'\theta\gamma-1)}{2}\frac{\partial b_l'}{\partial \tilde{\tau}_l}\Big)\end{aligned} \tag{96}$$

$$\begin{aligned}\frac{\partial b_l'}{\partial \tilde{\tau}_l} &= \frac{1}{4\sqrt{1-\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tilde{\tau}_l]}}}\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tilde{\tau}_l]^2}\frac{(E[\tau^2|\tau\geq\tilde{\tau}_l]-\tilde{\tau}_l^2)f(\tilde{\tau}_l)}{1-F(\tilde{\tau}_l)} \\ &= \frac{1}{4(2b_l'-1)}\frac{E[\tau^2]}{E[\tau^2|\tau\geq\tilde{\tau}_l]^2}\frac{(E[\tau^2|\tau\geq\tilde{\tau}_l]-\tilde{\tau}_l^2)f(\tilde{\tau}_l)}{1-F(\tilde{\tau}_l)}\end{aligned}. \tag{97}$$

Hence, $\frac{\partial B}{\partial \tilde{\tau}_l}$ becomes:

$$\frac{\partial B}{\partial \tilde{\tau}_l} = 2\tilde{\tau}_l\gamma\theta\Big(b_l'(1-b_l'\gamma\theta) - \frac{(2b_l'\theta\gamma-1)\tilde{\tau}_l}{8(2b_l'-1)}\frac{E[\tau^2]}{E[\tau^2 \mid \tau\geq\tilde{\tau}_l]^2}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l}\Big) \tag{98}$$

Since $b'(1-b') = \frac{E_F[\tau^2]}{4E_G[\tau^2|\tau\geq\tilde{\tau}_l]}$ and $b'(1-\gamma\theta b') \geq \hat{b}(1-\hat{b})$ for $\gamma\theta \leq 1$:

$$\begin{aligned}\frac{\partial B}{\partial \tilde{\tau}_l} &\geq 2\tilde{\tau}_l\gamma\theta\Big(\frac{E_F[\tau^2]}{4E_G[\tau^2|\tau\geq\tilde{\tau}_l]} - \frac{(2b_l'\gamma\theta-1)\tilde{\tau}_l}{8(2b_l'-1)}\frac{E_F[\tau^2]}{E_G[\tau^2|\tau\geq\tilde{\tau}_l]^2}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l}\Big) \\ &\geq \frac{E_F[\tau^2]\tilde{\tau}_l\gamma\theta}{2E_G[\tau^2|\tau\geq\tilde{\tau}_l]}\Big(1-\frac{(2b_l'\gamma\theta-1)\tilde{\tau}_l}{2(2b_l'-1)}\frac{1}{E_G[\tau^2|\tau\geq\tilde{\tau}_l]}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l}\Big) \\ &\geq \frac{E_F[\tau^2]\tilde{\tau}_l\gamma\theta}{2E_G[\tau^2|\tau\geq\tilde{\tau}_l]}\Big(1-\frac{1}{2}\frac{\tilde{\tau}_l}{E_G[\tau^2|\tau\geq\tilde{\tau}_l]}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l}\Big)\end{aligned}. \tag{99}$$

T2' implies

$$\Big(1-\frac{1}{2}\frac{\tilde{\tau}_l}{E[\tau^2 \mid \tau\geq\tilde{\tau}_l]}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l}\Big) \geq 0,$$

Therefore $\dfrac{\partial B}{\partial \tilde{\tau}_l} \geq 0$ and a unique maximum exists at $\tau^* = \overline{\tau}$. If $\gamma\theta > 1$, we have either $\frac{\partial B}{\partial \tilde{\tau}_l} \geq 0$ or $\frac{\partial B}{\partial \tilde{\tau}_l} < 0$. Suppose $\frac{\partial B}{\partial \tilde{\tau}_l} < 0$. Define

$$Z(\theta\gamma) = b_l'(1-b_l'\gamma\theta) - \frac{(2b_l'\theta\gamma-1)\tilde{\tau}_l}{8(2b_l'-1)}\frac{E[\tau^2]}{E[\tau^2 \mid \tau\geq\tilde{\tau}_l]^2}\frac{\partial E[\tau^2|\tau\geq\tilde{\tau}_l]}{\partial \tilde{\tau}_l} \tag{100}$$

Then

$$\frac{\partial B}{\partial \tilde{\tau}_l \partial\theta\gamma} = 2\tilde{\tau}_l\frac{\partial B}{\partial \tilde{\tau}_l} + 2\tilde{\tau}_l\gamma\theta\frac{\partial Z(\theta\gamma)}{\partial\gamma\theta} < 0, \tag{101}$$

which means that once $\frac{\partial B}{\partial \tilde{\tau}_l}$ becomes negative, then it can only get more negative as $\theta\gamma$ increases, if $\theta\gamma > 1$. Thus τ^* is either a unique interior maximizer or $\tau^* = \overline{\tau}$.

9.4. Proof of Proposition 11

We first determine the equilibrium values for $K_h(\tilde{\tau}_h)$ and $K_l(\tilde{\tau}_l)$. Define $H(\tilde{\tau}_i) = \frac{\delta(1-F(\tilde{\tau}_i))}{1+\delta(1-F(\tilde{\tau}_i))}$. The equilibrium salary in an h-firm is

$$w_h(\tilde{\tau}) = \gamma\theta E[\tau \mid \tau \geq \tilde{\tau}] - \gamma\theta\mu - H(\tilde{\tau}_i)(\theta E[\tau \mid \tau \geq \tilde{\tau}_h] - w_h(\tilde{\tau}_h) - \theta\gamma\mu) \tag{102}$$

or

$$w_h(\tilde{\tau}_h) = \theta\frac{\gamma - H(\tilde{\tau}_h)}{1 - H(\tilde{\tau}_h)}E[\tau \mid \tau \geq \tilde{\tau}_h] - \theta\gamma\mu, \tag{103}$$

which implies

$$K_h(\tilde{\tau}_h) = H(\tilde{\tau}_h)\theta\left(E[\tau \mid \tau \geq \tilde{\tau}_h] - \frac{\gamma - H(\tilde{\tau}_h)}{1 - H(\tilde{\tau}_h)}E[\tau \mid \tau \geq \tilde{\tau}_h] + \gamma\mu - \gamma\mu\right) \tag{104}$$

which simplifies to

$$K_h(\tilde{\tau}_h) = \theta\delta(1-\gamma)\int_{\tilde{\tau}_h}^{\overline{\tau}} \tau dF(\tau), \tag{105}$$

and we then have

$$K_l(\tilde{\tau}_l) = H(\tilde{\tau}_l)(E[\tau \mid \tau \geq \tilde{\tau}_l] - \theta\gamma E[\tau \mid \tau \geq \tilde{\tau}_l] + \theta\gamma\mu + K_h(\tilde{\tau}_h) - \gamma\mu) \tag{106}$$

$$K_l(\tilde{\tau}_l) = H(\tilde{\tau}_l)[(1-\theta\gamma)E[\tau \mid \tau \geq \tilde{\tau}_l] + (\theta - 1)\gamma\mu + K_h(\tilde{\tau}_h)]. \tag{107}$$

If. The existence and uniqueness of the threshold for an h-firm is proven in Proposition 1.

The threshold for an l-firm is given by:

$$\tilde{\tau}_l - \max\{\gamma\theta(E[\tau \mid \tau \geq \tilde{\tau}_i] - \mu) - K_h(\tilde{\tau}_h), 0\} - \gamma\mu - K_l(\tilde{\tau}_l) = 0. \tag{108}$$

For $\tilde{\tau}_l = \underline{\tau}$, we have have $\underline{\tau} - \gamma\mu - K_l(\tilde{\tau}_l) < 0$.

$$K_l(\underline{\tau}) = \frac{\delta}{1+\delta}[(1-\theta\gamma)\mu + (\theta-1)\gamma\mu + K_h(\tilde{\tau}_h)] = \frac{\delta}{1+\delta}[(1-\gamma)\mu + K_h(\tilde{\tau}_h)] \tag{109}$$

For $\tilde{\tau}_l = \overline{\tau}$, $K_l(\overline{\tau}) = 0$ and we have $\tilde{\tau}_l - \gamma\theta(\overline{\tau} - \mu) - \gamma\mu + K_h(\tilde{\tau}_h) > 0$,which is equivalent to $\theta\gamma < \frac{\overline{\tau} - \gamma\mu + K_h(\tilde{\tau}_h)}{\overline{\tau} - \mu}$. Therefore a solution exists, provided this condition holds.

To prove the uniqueness of the equilibrium we need to prove that the left-hand side of

(16) increases with $\tilde{\tau}_l$:

$$1-\theta\gamma\frac{\partial E[\tau\mid\tau\geq\tilde{\tau}_l]}{\partial\tilde{\tau}_l}-\frac{\partial K_l\left(\tilde{\tau}_l\right)}{\partial\tilde{\tau}_l}>0 \tag{110}$$

We have that

$$\frac{\partial K_l\left(\tilde{\tau}_l\right)}{\partial\tilde{\tau}_l} = \frac{\partial H\left(\tilde{\tau}_l\right)}{\tilde{\tau}_l}\left[(1-\theta\gamma)\,E[\tau\mid\tau\geq\tilde{\tau}_l]+(\theta-1)\,\gamma\mu+K_h\left(\tilde{\tau}_h\right)\right]+ \tag{111}$$

$$(1-\theta\gamma)\,\frac{\partial E[\tau\mid\tau\geq\tilde{\tau}_l]}{\partial\tilde{\tau}_l}H\left(\tilde{\tau}_l\right), \tag{112}$$

Thus we have

$$1-\theta\gamma\frac{\partial E[\tau\mid\tau\geq\tilde{\tau}_l]}{\partial\tilde{\tau}_l}-\frac{\partial K_l\left(\tilde{\tau}_l\right)}{\partial\tilde{\tau}_l} \tag{113}$$

$$= (1-\theta\gamma)\left[1-H\left(\tilde{\tau}_l\right)\right]\frac{\partial E[\tau\mid\tau\geq\tilde{\tau}_l]}{\partial\tilde{\tau}_l}-\frac{\partial H\left(\tilde{\tau}_l\right)}{\partial\tilde{\tau}_l}\frac{K_l\left(\tilde{\tau}_l\right)}{H\left(\tilde{\tau}_l\right)}>0 \tag{114}$$

due to Assumption T2 and the fact that $H\left(\tilde{\tau}_l\right)<1$ and $\frac{\partial H(\tilde{\tau}_l)}{\partial\tilde{\tau}_l}\leq 0$.

Only if. The same as in Proposition 1.

References

An, M. Y. 1998. Logconcavity versus Logconvexity: A Complete Characterization. Journal of Economic Theory 80, 350-369.

Burdett, K. 1996. Truncated Means and Variances. Economics Letters 52, 263-267.

Edmans, A., and X. Gabaix. 2011. The Effect of Risk on the CEO Market. Review of Financial Studies 24, 2822-2863.

Edmans, A., Gabaix, X., and A. Landier. 2009. A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium. Review of Financial Studies 22, 4881-4917.

Eisfeldt, A. L., and C. M. Kuhnen. 2011. CEO Turnover in a Competitive Assignment Framework. UCLA and Northwestern U., working paper.

Frydman, C. 2005. Rising through the ranks: The evolution of the market for corporate executives, 1936-2003. MIT Sloan, Working Paper.

Frydman, C., and D. Jenter. 2010. CEO Compensation. Annual Review of Financial Economics 2: 75-102

Frydman, C, and R. Saks. 2010. Executive Compensation: A New View from a Long-Run Perspective, 1936-2005. Review of Financial Studies 23: 2099-2138

Gabaix, X., and A. Landier. 2008. Why has CEO pay increased so much? Quarterly Journal of Economics 123, 49–100.

Murphy, K. J., and J. Zabojnik. 2006. Managerial Capital and the Market for CEOs. Queen's University, working paper.

Tervio, M. 2008. The difference that CEOs make: An assignment model approach. American Economic Review 98, 642–668.