Financing Through Asset Sales^{*}

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Abstract

Most existing research on firm financing studies the choice between debt and equity issuance. This paper analyzes an alternative source – selling non-core assets. We identify three new factors that drive a firm's choice between asset sales and equity issuance. First, investors in an equity issue share in the cash raised. Since the value of cash is certain, this mitigates the information asymmetry of equity (the "certainty effect"). In contrast to Myers and Majluf (1984), even if non-core assets are less volatile, the firm issues equity if the financing need is high. This result is robust to using the cash raised to finance an uncertain investment. Second, if non-core assets are (dis)synergistic, this drives a firm towards selling assets (equity). This "synergy effect" interacts with the certainty effect. When financing needs rise, high-types may sell equity even if assets are dissynergistic. Third, selling equity implies a "lemons" discount not only for the equity being issued but also for the rest of the firm, since its value is perfectly correlated with the issued equity. In contrast, even if an asset seller suffers a "lemons" discount for the disposed asset, this need not lead to a low stock price as the asset is not a carbon-copy of the rest of the firm (the "correlation effect").

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One of the most important decisions that a firm faces is how to raise financing. Most existing research focuses on the choice between debt and equity financing, with various theories identifying different factors that drive a firm's security issuance decision. The trade-off theory argues that managers compare the benefits of debt (tax shields and a reduction in the agency costs of equity) against its costs (bankruptcy costs and the agency costs of debt). The pecking-order theory of Myers (1984), motivated by the model of Myers and Majluf (1984, "MM"), posits that managers issue securities that exhibit least information asymmetry. The market timing theory of Baker and Wurgler (2002) suggests that managers sell securities that are most overpriced by the market.

While there is substantial research on financing through security issuance, another major source of financing is relatively unexplored – selling non-core assets such as a division or a plant. Asset sales are substantial in reality: in 2010, Securities Data Corporation (SDC) records \$133 billion in asset sales in the US^1 , compared to \$130 billion in seasoned equity issuance. Although some of these sales may have been motivated by business reasons, capital raising is an important driver of many disposals. Major firms in the oil and gas industry (including Chevron, Shell, and Conoco) have recently sold non-core divisions to raise capital for liquidity and debt service. Most notably, in October 2011, BP set a target of \$45 billion in asset sales to cover the costs of the Deepwater Horizon spill. Banks worldwide have raised billions of dollars through asset sales in the recent crisis to reassure investors, replenish depleted capital, and build capital in anticipation of new regulatory standards.² More broadly, Campello, Graham, and Harvey (2010) finds that 70% (37%) of financially constrained (unconstrained) firms increased their asset sales in the financial crisis. This difference points to asset sales being used as a financing tool. Ofek (1993) and Asquith, Gertner, and Scharfstein (1994) show that firms sell assets in response to financial constraints, and Jain (1985) and Lang, Poulsen, and Stulz (1995) document that asset sales follow poor firm-level performance. Borisova, John, and Salotti (2011) examine the press announcements to asset sales and find that over half of sellers state financing motives. Hite, Owers, and Rogers (1987) examine the stated motives for asset sales and note that "in several cases, management indicated that assets were being sold to raise capital for expansion of existing lines of business or to reduce high levels of debt. In other words, selling assets was viewed as an alternative to the sale of new securities."

¹This figure is an underestimate as 60% of asset sales in SDC have missing transaction values.

²In September 2011, Banque Nationale de Paris and Société Générale announced plans to raise \$96 billion and \$5.4 billion respectively through asset sales, to create a financial buffer against contagion from other French banks. Bank of America raised \$3.6 billion in August 2011 by selling a stake in a Chinese construction bank, and \$755 million in November 2011 from selling its stake in Pizza Hut.

This paper analyzes the role of asset sales as a means of financing. In particular, it studies the conditions under which asset sales are preferable to equity issuance. We build a deliberately parsimonious model to maximize tractability; this allows for the key expressions to be solved for in closed form and the economic forces driving the results to be transparent. The firm comprises a core asset and a non-core asset. The firm must raise financing to meet a liquidity need, and can either issue equity, which is a claim on the entire firm, or sell part of the non-core asset. Following MM, we model information asymmetry as the principal driver of the financing decision. The firm's type is privately known to its manager and comprises two dimensions. The first dimension is quality, which determines the assets' standalone (common) values. The value of the core asset is higher for high-quality firms. The value of the non-core asset depends on how we specify the correlation between the core and non-core assets. With a positive (negative) correlation, the value of the non-core asset is higher (lower) for high-quality firms. The second dimension is synergy, which determines the additional (private) value that the non-core asset is worth to its current owner. The incorporation of a synergy dimension means that asset sales may occur for either financing or business reasons.

It may seem that asset sales can already be analyzed by applying the general principles of the MM model of security issuance to the sale of assets, removing the need for a new theory specific to asset sales. Such an extension would suggest that assets are preferred to equity if they exhibit less information asymmetry. While information asymmetry is indeed an important consideration, our model identifies several new distinctions between asset sales and equity issuance that also drive the financing choice, and may swamp information asymmetry considerations.

First, an advantage of equity issuance is that new shareholders obtain a stake in the entire firm. This includes not just the core and non-core assets in place (whose value is uncertain), but also the cash paid for the new shares. Since the value of this cash is certain, this mitigates the information asymmetry associated with the assets in place: the *certainty effect*. In contrast, the purchaser of a non-core asset does not share in the cash raised, and thus bears the full information asymmetry associated with the asset's value. Hence, in contrast to MM, even if the non-core asset is less volatile than the firm as a whole, the manager may sell equity if enough cash is raised that the certainty effect outweighs the higher information asymmetry of equity. Formally, a pooling equilibrium is sustainable where all firms sell assets (equity) if the financing need is sufficiently low (high). Since the certainty effect strengthens as the amount of financing increases, the model delivers the new result that the firm's financing choice depends on the amount of financing required. This dependence contrasts standard financing models, where the choice depends only on inherent characteristics of the security (such as its level of information asymmetry (MM) or misvaluation (Baker and Wurgler (2002)), and not the level of financing required – unless one assumes exogenous frictions such as transactions costs or limits on the amount of financing that can be raised through a given channel (e.g., notions of debt capacity). In our model, the level of financing needs are a motive for selling assets, it may seem that a greater financing need will lead to more assets being sold. In contrast, we show that it may reduce the volume of assets traded, as the firm substitutes into equity. Thus, a greater financial shock can improve real efficiency, as firms hold onto synergistic assets and instead sell equity.

The certainty effect applies for any use of cash whose value is uncorrelated with firm quality: for example, holding cash on the balance sheet or using it to repay debt or pay dividends. We extend the model to allow the cash to be used to finance an uncertain investment, whose expected value is correlated with firm quality. It may appear that this extension should weaken the certainty effect, since the funds raised are no longer held as certain cash. This intuition turns out to be incomplete, because there is a second effect. Since investment is positive-NPV, it increases the value of the capital that investors contribute to the firm. If the desirability of investment (for firms of both quality) is sufficiently high compared to the additional investment return generated by the high-quality firm over the low-quality firm, the second effect dominates - somewhat surprisingly, the certainty effect can strengthen when cash is used to finance an uncertain investment. Thus, equity issuance becomes easier to sustain compared to the core model. In contrast, if investment is sufficiently volatile, the first effect is stronger and asset sales become preferable. Thus, the source of financing depends on the use of financing, even though we have a model of pure adverse selection with neither moral hazard nor bankruptcy costs. In all scenarios, it remains the case that asset (equity) sales are used for low (high) financing needs.

A second consideration is the synergies between the non-core asset and the rest of the firm. This *synergy effect* leads to "threshold" separating equilibria in which a firm sells assets if synergies are below a cutoff, and issues equity otherwise. While the idea that firms are less willing to sell assets if they are synergistic is unsurprising, of greater interest is how this synergy effect interacts with the certainty effect. The threshold synergy level is different for high- and low-quality firms. If the amount of financing required increases, this augments the certainty affect and reduces the information asymmetry of equity. This makes equity issuance more (less) attractive to high (low) quality firms: even if its assets are dissynergistic, a high-quality firm chooses to retain them and instead issue equity, because its suffers a smaller adverse selection discount. Thus, higher financing needs mean that lower-quality assets are traded, and thus reduce the market price of assets. Conversely, it increases both the quality and price of equity sold in equilibrium.

A third difference is the *correlation effect*, which represents an advantage to selling assets. When a firm issues equity, it suffers an Akerlof (1970) "lemons" discount on the equity issued – the market infers that the equity issued is low-quality, from the firm's decision to issue it. This leads to the market not only paying a low price for the new equity issued, but also attaching a low valuation to the remainder of the firm. This is because the new equity being issued and the equity of the firm as a whole are necessarily perfectly correlated, since the former is a carbon copy of the latter. In contrast, when a firm sells non-core assets, it receives a low price, but critically this need not imply a low valuation for the rest of the firm as the asset sold need not be a carbon copy – indeed, it may be negatively correlated.

If synergies are sufficiently weak, a negative correlation allows for a separating equilibrium where firms separate by quality alone and not by synergies: all high-quality firms sell assets and all low-quality firms issue equity. Even though the high-quality firm receives a (fair) low price for the assets he sells, so there are no market timing motives, this low price for assets does not imply a low valuation for the rest of the firm, due to the correlation effect. Thus, firms can sell poorly-performing assets without suffering negative inferences on the company as a whole. If synergies are strong, the separating equilibrium is a threshold equilibrium, where firms separate by both quality and synergies. This is similar to the positive correlation case, but has a number of additional features. First, the threshold synergy level (below which firms sell assets) is always higher for high-quality firms: since their assets are less valuable, they are more willing to sell them. Second, high-quality firms can make a capital gain from selling their low-quality assets. In a model of information asymmetry alone, the market would correctly infer that an asset seller has low-quality assets and correctly ascribe a low price, eliminating the possibility of a capital gain. However, with a synergy motive, a firm is able to disguise an asset sale motivated by overvaluation reasons (the asset is of

low quality and thus has a low common value) as instead being motivated by business reasons (the asset is dissynergistic and thus only has a low private value).

As with the positive correlation case, there may be an asset-pooling equilibrium. This equilibrium is relatively easy to sustain, as a deviation to equity issuance leads not only to a low price for the equity being sold, but also a low valuation on the rest of the firm. The equity-pooling equilibrium is harder to sustain due to the correlation effect: even if deviation leads to a low price for the asset sold, it does not imply a low valuation for the rest of the firm. Formally, the parameter values under which the equity-pooling equilibrium is sustainable form a strict subset of those under which the asset-pooling equilibrium is sustainable.

Our paper can be interpreted more broadly as showing at what *level* to issue claims for financing reasons: the firm level (equity issuance) or the asset level (asset sales). Many of the effects also apply to other *types* of claim that the firm can issue at each level. All three effects apply to parent-company debt in the same way as parentcompany equity: since debt is also a claim to the entire firm, it also benefits from the certainty effect and is positively correlated with firm value; issuing debt does not involve the loss of synergies. Thus, our model abstracts from debt financing and focuses on the choice between selling assets and equity. Similarly, if a firm issues collateralized debt at the asset/division level, or engages in an equity carve-out of the division, this need not imply low quality for the firm as a whole (correlation effect), and investors do not own a claim to the cash that they invest, which resides at the parent company level (certainty effect). The one difference versus an asset sale is that issuing asset-level debt or equity does not involve a loss of (dis)synergies: thus, a carve-out can be seen as a special case of our model where synergies are zero.³ *[Perhaps it's for fut res]* Thus, the model's main forces more generally apply to the *level* at which to issue claims, rather than the type of claim. If these effects suggest that it is optimal to issue claims at the parent company level, then the standard pecking-order theory can be used to analyze if the type of claim should be parent-company debt or parent-company equity.

Most of the existing literature on asset sales is empirical. Jain (1985), Klein (1986), Hite, Owers, and Rogers (1987), and Slovin, Sushka, and Ferraro (1995), among others, find positive market reactions to asset sales. Lang, Poulsen, and Stulz (1995) show that this positive reaction stems from financing rather than business reasons. Brown, James, and Mooradian (1994) and Bates (2006) examine the use of proceeds from an asset sale. Maksimovic and Phillips (2001) analyze business reasons rather than financing motives.

³In a future draft we will also analyze the case in which synergies are non-zero and the firm has a choice between asset sales, carve-outs, and equity issuance.

Existing theories consider asset sales as the only source of financing and do not compare it to equity issuance: a partial list includes Shleifer and Vishny (1992), De-Marzo (2005), He (2009), and Kurlat (2010). Milbradt (2012) and Bond and Leitner (2011) show that selling an asset will affect the market price of the seller's remaining portfolio under mark-to-market accounting, changing his balance sheet constraint. We show that such correlation effects are stronger for equity issuance: while a partial asset sale may imply a negative valuation of the remaining unsold non-core assets, it need not imply a negative valuation of the whole firm. Nanda and Narayanan (1999) also consider both asset sales and equity issuance under information asymmetry. They do not feature the certainty, synergy, or correlation effects that are the heart of this paper: they only consider the case of negative correlation, and the manager does not care about the stock price so there is no correlation effect. They instead focus on other interesting issues such as optimal firm scope.⁴

Since a partial asset sale in the no-synergies case can also be interpreted as a carveout, our paper is also related to the literature on carve-outs. Nanda (1991) also points out that non-core assets may be uncorrelated with the core business and that this may motivate a firm to issue equity at the subsidiary level. In his model, non-core assets always have a zero correlation, and the information asymmetry of core and noncore assets is the same. Our model allows for general correlations and information asymmetries, thus enabling us to generate the certainty and correlation effects. ⁵

The remainder of the paper is organized as follows. Section 1 outlines the general model. Sections 2 and 3 study the positive and negative correlation cases respectively. In Section 4, the funds raised are used to finance an uncertain investment, and Section 5 concludes. Appendix A contains all proofs not in the main text. In the Online Appendix, Appendix B contains additional comparative statics, and Appendices C and E contain other peripheral material.

⁴Leland (1994) allows firms to finance cash outflows either by equity issuance (in the core analysis) or by asset sales (in an extension), but not to choose between the two. In Strebulaev (2007), asset sales are assumed to be always preferred to equity issuance, which is a last resort. Other papers model asset sales as a business decision (equivalent to disinvestment) and do not feature information asymmetry. In Morellec (2001), asset sales occur if the marginal product of the asset is less than its (exogenous) resale value. In Bolton, Chen, and Wang (2011), disvinestment occurs if the cost of external finance is high relative to the marginal productivity of capital. While those papers take the cost of financing as given, this paper microfounds the determinants of the cost of equity finance versus asset sales.

⁵Empirically, Allen and McConnell (1998) study how the market reaction to carve-outs depends on the use of proceeds. Schipper and Smith (1986) show that equity issuance leads to negative abnormal returns, but carve-outs lead to positive returns. Slovin, Sushka, and Ferraro (1995) find positive market reactions to carve-outs, and Slovin and Sushka (1997) study the implications of parent and subsidiary equity issuance on the stock prices of both the parent and the subsidiary.

1 The Model

The model consists of two types of risk-neutral agents: firms, which raise financing, and investors, who provide financing and set prices. The firm is run by a manager, who has private information about the firm's type $\theta = (q, k)$. If a firm is of type θ , we also say that the manager is of type θ . The type θ consists of two dimensions. The first is the firm's quality $q \in \{H, L\}$, which measures the standalone (common) value of its assets. The prior probability that q = H is $\pi \geq \frac{1}{2}$, where higher π corresponds to lower information asymmetry. The second dimension is a synergy parameter $k \sim U[\underline{k}, \overline{k}]$, where $\underline{k} \leq 0$, $\overline{k} \geq 0$, and k and q are uncorrelated. This parameter measures the additional (private) value created by the existing owner.

The firm comprises two assets. The core business has value C_q , where $C_H > C_L$, and the non-core business has value A_q .⁶ Where there is no ambiguity, we will use the term "assets" to refer to the non-core business. We consider two specifications of the model. The first is $A_H > A_L$, so that the value of the two assets is positively correlated. The second is $A_L > A_H$, so the assets are negatively correlated. In both cases, we assume that:

$$C_H + A_H > C_L + A_L,\tag{1}$$

i.e. H has a higher total value. Thus, even if assets are negatively correlated, the higher value of L's non-core assets is outweighed by the lower value of its core assets. In Myers (1984), the key driver of financing choice is the volatility of the security being issued. The distinction between the two cases of $A_H > A_L$ and $A_H < A_L$ shows that it is not only the volatility of the non-core asset that matters ($|A_H - A_L|$), but also its correlation with the core asset ($sign (A_H - A_L)$).⁷ Note that the volatility of a claim can also be referred to as its information asymmetry, since the manager knows the true value but investors do not.

We consider the financing decision of an individual probabilistic firm. The firm must raise financing of $F < \min(A_L, A_H)$.⁸ This need could arise from a number of sources: a liquidity need (e.g. providing a buffer against future cash requirements),

⁶The non-core asset can be considered either a physical asset (e.g. a plant) or a financial asset (e.g. an investment in another firm).

⁷If $A_H = A_L$, there is no information asymmetry surrounding the non-core asset. Thus, it is automatic that the firm will always raise financing by selling it (as shown by MM).

⁸The amount of financing F does not depend on the source of financing: F must be raised regardless of whether the firm sells assets or equity. In bank capital regulation, equity issuance leads to a superior improvement in capital ratios than asset sales and so F does depend on the source of financing. We do not consider this effect here as the effect will be straightforward: it will encourage firms towards the source that reduces the amount of financing required.

an upcoming debt repayment, or a dividend payment. The cash raised remains on the firm's balance sheet, which nests any use of cash that increases equity value by an amount F in expectation, independent of firm quality (which includes all the above cases). In Section 4 we allow the cash to fund an investment whose value is correlated with firm quality, so that there is information asymmetry on the use of the cash.

We currently treat the firm's financing need F as exogenous. In MM, the firm has the option not to raise financing and instead to forgo investment; the goal of that paper is to show that information asymmetry can deter investment by hindering financing. Since the trade-off between forgoing a desirable investment, and suffering an adverse selection discount when raising financing for investment, is already well-known, our focus instead is to study the choice between asset sales and equity to meet a given financing need, and so we take F as given. In ongoing work we are extending the model to give firms the choice of whether to raise financing and for financing needs to be privately known; preliminary results are described in the Conclusion.

The firm can raise funds either by selling non-core assets or by issuing equity; it cannot sell the core asset as it is essential for the firm. (In the Conclusion, we discuss an extension in which we relax this assumption.) Since $F < \min(A_L, A_H)$, it is possible to raise the required financing entirely through either source. We restrict attention to equilibria in which the firm is assumed to raise financing from a single source. This can be motivated by transactions costs associated with using multiple financing sources. (Appendix C formally derives conditions under which the firm will not wish to deviate to multiple financing sources.) There are no taxes, and any transactions costs are assumed to be the same for both asset sales and equity issuance.

The non-core asset is perfectly divisible so partial asset sales are possible. We deliberately do not feature nonlinearities as they will mechanically generate the result that the source of financing depends on the amount of financing required. If a firm sells non-core assets with a value of X, its fundamental value falls by X(1+k). Thus, the case of k > (<) 0 represents synergies (dissynergies), where the non-core asset is worth more (less) to the current owner than in its next-best use.⁹

Formally, a firm of type θ issues a claim $K_{\theta} \in \{E, A\}$, where $K_{\theta} = E$ represents equity issuance and $K_{\theta} = A$ an asset sale. Investors infer the firm's type based on its

⁹One may wonder why the firm will have dissynergistic assets to begin with. First, even if the asset is dissynergistic, the firm may retain it due to the transactions costs of asset sales: only if it is forced to raise financing and so would have to bear the transactions costs of equity issuance otherwise would it consider selling assets. Second, the market for assets is not perfectly frictionless, and so not all assets are owned by the best owner at all times. Our model allows for $\underline{k} = 0$ in which case there are no dissynergies.

choice of claim K_{θ} . These inferences affect both the firm's current market valuation (also referred to as its stock price) and the terms at which it raises financing. Investors are perfectly competitive and price the claim being sold at its expected value.¹⁰ Thus, the firm may enjoy either an increase or decrease in its fundamental value, depending on whether it issues the claim at a gain or a loss. The manager's objective function places weight ω on the firm's stock price and $1-\omega$ on fundamental value. If a manager is indifferent between playing the equilibrium strategy and deviating, we assume that he remains with the equilibrium strategy. A useful feature of the framework is that only the quality parameter q, and not the synergy parameter k, affects the price that investors pay for claims. This allows our model to incorporate two dimensions of firm type – quality and synergy – while retaining tractability. Thus, we will sometimes use the term "H" or "H-firm" to refer to a high-quality firm regardless of its synergy parameter, and similarly for "L" or "L-firm". We will use the terms "capital gain" and "capital loss" to refer to the gain or loss resulting from the common value component of the asset value only, and "fundamental gain" and "fundamental loss" to refer to the overall change in the firm's fundamental value from selling assets. The latter consists of both the capital gain/loss and the loss of (dis)synergies by trading the asset. For equity issuance, the capital gain/loss equals the fundamental gain/loss.

We use the Perfect Bayesian Equilibrium ("PBE") solution concept, which involves the following: (i) Investors have a belief about which manager types issue which claim K_{θ} . (ii) The price of the claim being issued equals its expected value, conditional on investors' beliefs in (i). (iii) Each manager type chooses to issue the claim K_{θ} that maximizes his objective function, given investors' beliefs. (iv) Investors' beliefs satisfy Bayes' rule. In addition to the PBE, beliefs on claims K_{θ} issued off the equilibrium path satisfy the Cho and Kreps (1987) Intuitive Criterion (IC).

We first analyze the positive correlation version of the model $(A_H > A_L)$ and then move to the negative correlation version $(A_L > A_H)$.

2 Positive Correlation

We assume that $\omega = 0$ in this section for ease of exposition. The role of $\omega > 0$ only exists under negative correlation, where there is a trade-off to being inferred as L: market valuation falls, but the firm receives a high price if it sells assets. With a

¹⁰Sicherman and Pettway (1987), John and Ofek (1995) and Datta, Iskandar-Datta and Raman (2003) find insignificant returns to purchasers in asset sales.

positive correlation, there is no such trade-off: being inferred as quality L is detrimental for both market valuation and fundamental value, and so $\omega > 0$ does not affect the sustainability of any equilibria.

We first consider the potential pooling equilibria, which are of two types: an assetpooling equilibrium (APE) and an equity-pooling equilibrium (EPE). We then move to separating equilibria (SE).

2.1 Pooling Equilibrium, All Firms Sell Assets

We consider a pooling equilibrium in which all firms sell assets, supported by the offequilibrium path belief (OEPB) that anyone who sells equity is of quality L. Assets are valued at

$$E[A] = \pi A_H + (1 - \pi)A_L.$$
 (2)

If equity is sold (off the equilibrium path), it is valued at

$$C_L + A_L + F. \tag{3}$$

The additional F term arises because the cash raised from financing enters the balance sheet, and so new shareholders own a claim to this cash in addition to the two existing assets.¹¹

The fundamental value of H and L are respectively given by:

$$C_{H} + A_{H} - F \frac{(1-\pi)(A_{H} - A_{L}) + kA_{H}}{E[A]}, \qquad (4)$$

$$C_{L} + A_{L} + F \frac{\pi (A_{H} - A_{L}) - kA_{L}}{E[A]}.$$
(5)

An *L*-firm enjoys a capital gain of $\frac{\pi F(A_H - A_L)}{E[A]}$ by selling low-quality assets at a pooled price. However, it also loses the synergies from the asset. If:

$$1 + \overline{k} < \frac{E[A]}{A_L},\tag{6}$$

then even the *L*-firm with the greatest synergies, type (L, \overline{k}) , will prefer to sell assets rather than to deviate, since the capital gain from selling low-quality assets exceeds

¹¹This is consistent with the treatment of financing in MM, although the level of financing plays no role in their analysis since both equity and debt are claims on the entire firm, which includes the financing raised.

the loss of synergies. We assume that equation (6) holds throughout the case of positive correlation, else synergy motives are so strong that they dominate information asymmetry considerations. Equation (6) is a necessary and sufficient condition for all *L*-firms not to deviate, regardless of F.

H-firms suffer a capital loss of $\frac{(1-\pi)F(A_H-A_L)}{E[A]}$ in addition to any loss of synergies. A *H*-firm may thus deviate and issue equity. If it does so, fundamental value becomes:

$$C_H + A_H - \frac{F(C_H + (1+k)A_H - C_L - A_L)}{C_L + A_L + F}.$$
(7)

The no-deviation ("ND") condition is that $(7) \leq (4)$. This condition is most stringent for type (H, \overline{k}) . Thus, all *H*-firms will not deviate if:

$$F \le F^{APE,ND,H} = \frac{(C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \overline{k})}{A_H(1 + \overline{k}) - E[A]}.$$
(8)

This condition is equivalent to the "unit cost of financing" being lower for asset sales, i.e.

$$\frac{A_H\left(1+\overline{k}\right)}{E[A]} \le \frac{C_H + A_H + F}{C_L + A_L + F},\tag{9}$$

where the numerator on each side is the value of the claim being sold to the firm, and the denominator is the price that investors pay for that claim.

There are three forces that determine H's incentives to deviate. The first is whether equity or assets are more volatile. This effect is a natural extension of the MM principle that high-quality firms wish to issue safe claims. Indeed, if there are no synergy considerations (i.e. $\overline{k} = 0$), then if $\frac{A_H}{E[A]} > \frac{C_H + A_H}{C_L + A_L}$, i.e. assets are sufficiently more volatile than equity, then the RHS of (8) is negative and so APE is unsustainable for any F. H-firms suffer a smaller capital loss by issuing undervalued equity compared to selling undervalued assets, and so will deviate to equity issuance.

The second force is the synergy effect, which is absent in MM. For APE to be sustained, the type with the greatest synergy \overline{k} must be willing to sell assets despite the loss of synergies. Thus, sustainability requires not only assets to be sufficiently safe, but also the maximum synergy level to be sufficiently small. If $\overline{k} > \frac{(C_H + A_H)E[A]}{(C_L + A_L)A_H} - 1$, i.e. synergies are important (\overline{k} is high) and assets are risky ($\frac{A_H}{E[A]}$ is high compared to $\frac{C_H + A_H}{C_L + A_L}$), then again the RHS of (8) is negative and so APE is unsustainable for any F.

The third force is the amount of financing F being raised. This is unique to a model

of asset sales and stems from the fact that the cash raised from financing enters into the firm's balance sheet. Thus, if the investor purchases an equity claim, she shares in the value of this cash; but if she buys non-core assets, she does not. Since the value of cash is certain, this effect mitigates the information asymmetry associated with equity financing: the RHS of equation (9) becomes dominated by the term F, which is the same in the numerator and the denominator as there is no information asymmetry, and less dominated by the uncertain assets-in-place terms C_q and A_q . Thus, there is an upper bound on F to prevent deviation. If F exceeds this upper bound, the certainty effect is sufficiently strong that (H, \bar{k}) deviates to issuing equity – even though it is inferred as L by doing so. In particular, note that even if $\frac{A_H}{A_L} < \frac{C_H + A_H}{C_L + A_L}$ and $\bar{k} = 0$, i.e. assets are safer than equity and there are no synergies, it may be that (8) is violated so quality H issues equity. Thus, the MM result that the high type will issue the least volatile claim does not hold.

The effect of F on financing choices is interesting since it may seem that, since financing is a motive for asset sales, greater financing needs F should lead to greater asset sales. However, if F rises sufficiently, then the firm may end up selling *fewer* assets, since it is substituting into an alternative source of financing: equity issuance. Thus, surprisingly, greater financial constraints may improve real efficiency as firms hold onto their synergistic assets. (Equity issuance has no effect on real efficiency as it leads to a pure wealth transfer between investors and firms; asset sales affect real efficiency due to the difference between common and private values.) In particular, if $\underline{k} = 0$, all asset sales reduce total surplus since there are no dissynergies, and so higher F can increase total surplus. We now study the comparative statics on the upper bound (8). These are as follows:

$\partial F^{APE,ND,H}$	- $E[A] > 0$
∂C_H	$= \frac{1}{A_H(1+\bar{k}) - E[A]} > 0,$
$\partial F^{APE,ND,H}$	$=\frac{A_H(1+\overline{k})}{A_H(1+\overline{k})-E[A]}=1+\frac{\partial F^{APE,ND,H}}{\partial C_H}>0,$
$\partial(-C_L)$	
$\partial F^{APE,ND,H}$	$=\frac{E[A](1-\pi)(A_H-A_L)+\overline{k}E[A]-(1-\pi)A_L(C_H-C_L+A_H-A_L)(1+\overline{k})}{\overline{k}} \leq 0,$
∂A_H	$= \frac{(A_H(1+\overline{k}) - E[A])^2}{(A_H(1+\overline{k}) - E[A])^2} \ge 0,$
$\partial F^{APE,ND,H}$	$=\frac{A_{H}(1+\bar{k})\left[(1-\pi)(C_{H}-C_{L}+A_{H}-A_{L})+A_{H}(1+\bar{k})-E[A]\right]}{2} < 0.$
$\partial(-A_L)$	$(A_H(1+k) - E[A])^2$
$\partial F^{APE,ND,H}$	$=\frac{(C_H - C_L + A_H - A_L)A_H^2(1 + \overline{k})}{(A_H(1 + \overline{k}) - E[A])^2} > 0,$
$\partial \pi$	$= \frac{(A_H(1+\overline{k}) - E[A])^2}{(A_H(1+\overline{k}) - E[A])^2} > 0,$
$\partial F^{APE,ND,H}$	$= -\frac{A_H E[A](C_H - C_L + A_H - A_L)}{(A_H(1 + \overline{k}) - E[A])^2} < 0.$
$\partial \overline{k}$	$(A_H(1+\overline{k}) - E[A])^2 $

For the values of the core business, C_H and C_L , the signs of the derivatives are intuitive: increasing volatility $C_H - C_L$ (either by increasing C_H or reducing C_L) augments the loss that H makes by deviating to issue equity. This discourages deviation and the upper bound can relax, i.e., increase. The derivative with respect to $-C_L$ is larger because reducing C_L has an additional effect: it reduces the price H receives from deviating to sell equity $(C_L + A_L)$. He must therefore sell a greater fraction of the firm's equity, and so he bears the capital loss over a greater base (the "base effect").

Turning to the non-core asset, the negative sign of $\frac{\partial F^{APE,ND,H}}{\partial (-A_L)}$ arises because lowering A_L increases H's loss to selling assets for a pooled price, and thus encourages him to deviate to equity. This requires the upper bound on F to tighten, i.e. decrease. However, the sign of $\frac{\partial F^{APE,ND,H}}{\partial A_H}$ is ambiguous due to the base effect: an increase in A_L augments the pooled price that H receives from selling assets ($E[A] = \pi A_H + (1 - \pi)A_L$). This reduces the quantity of assets that the firm needs to sell and so the loss is sustained over a smaller base. The bound is also increasing in π . As H begins to dominate the market, he suffers a lower capital loss from pooling on asset sales, because the pooled price E[A] that he receives becomes closer to the true asset value of A_H . Thus, he has a lower incentive to deviate to selling equity.

Finally, the bound is decreasing in the maximum synergy \overline{k} . If \overline{k} is higher, (H, \overline{k}) is more willing to deviate to issue equity, and so F must fall to weaken the certainty effect. Viewed differently, even if \overline{k} is high (so that assets are synergistic), the firm will still be willing to sell them if F is sufficiently low. One might think that the effect

of synergies should be independent of the amount of financing raised: if assets are sufficiently synergistic, the firm will not sell them, regardless of F. However, we show that if F is low, the firm will be willing to sell assets, even if they are synergistic. The synergy effect interacts with the certainty effect.

We now verify whether the OEPB, that an equity issuer is of quality L, satisfies the IC. This is the case if an L-firm would weakly prefer to issue equity if revealed H. In this case, L's fundamental value from deviation is:

$$C_L + A_L + F - F\left(\frac{C_L + A_L + F}{C_H + A_H + F}\right).$$

Note that the price of equity in a deviation, (3), only requires the OEPB to be that a deviator is of quality L: it does not depend on the synergy parameter k. Thus, we can set the market's belief on the type of the deviator to be that which is most likely to deviate if revealed H, as this makes the IC easiest to satisfy. Since greater synergies increase the incentive to deviate, we set the OEPB to be that a deviator is of type (L, \overline{k}) . This firm type will deviate, satisfying the IC, if:

$$F \le F^{APE,IC} = \frac{(C_H + A_H)A_L(1 + \overline{k}) - E[A](C_L + A_L)}{E[A] - A_L(1 + \overline{k})}.$$
 (10)

It may seem that the IC should be trivial since L receives a high price for selling equity and being inferred as good, rather than a pooled price for selling assets. However, if F is large, selling equity is less attractive since the certainty effect reduces the gains from being inferred as H. Thus, we have another upper bound on F, again due to the certainty effect. If $\overline{k} < \frac{(C_L + A_L)E[A]}{(C_H + A_H)A_L} - 1$, i.e. assets are relatively volatile and synergies are small, the RHS of (10) is negative and so the IC is violated for any F. Type Lenjoys such a large capital gain from pooling on assets, and loses sufficiently small synergies, that he will not deviate to selling equity even if revealed as H.¹²

Lemma 1 below summarizes the equilibrium. The proof shows that the IC condition is stronger than the ND condition if and only if $1 + \overline{k} \leq \frac{E[A]}{\sqrt{A_H A_L}}$ (> 1). Thus, if this inequality is satisfied, the former is necessary and sufficient for the pooling equilibrium to hold, else the latter is the necessary and sufficient condition.

¹²To eliminate an equilibrium with $F > F^{APE,IC}$ via the IC, we also require that H will deviate if he is revealed good. This will automatically be the case, as he will break even rather than suffering a capital loss and losing synergies. In all of the other equilibria that we consider, it will similarly be automatic that H will deviate if he is revealed good, so we will not need to show this mathematically.

Lemma 1. (Positive correlation, pooling equilibrium, all firms sell assets.) Consider a pooling equilibrium in which all firms sell assets $(K_q = A)$ and a firm that issues equity is inferred as type (L, \overline{k}) , and assume that $\overline{k} < \frac{E[A]}{A_L} - 1$. The equilibrium is sustainable if $F \leq F^{APE}$, where

$$F^{APE} = \begin{cases} F^{APE,IC} = \frac{(C_H + A_H)A_L(1 + \bar{k}) - E[A](C_L + A_L)}{E[A] - A_L(1 + \bar{k})} & \text{if } 1 + \bar{k} \le \frac{E[A]}{\sqrt{A_H A_L}} \\ F^{APE,ND,H} = \frac{(C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - E[A]} & \text{if } 1 + \bar{k} > \frac{E[A]}{\sqrt{A_H A_L}}. \end{cases}$$
(11)

The upper bound $F^{APE,IC}$ is increasing in C_H , $-C_L$ and \overline{k} and decreasing in A_H , $-A_L$, and π . The upper bound $F^{APE,ND,H}$ is increasing in C_H , $-C_L$ and π , and decreasing in $-A_L$ and \overline{k} ; the effect of A_H is ambiguous.

2.2 Pooling Equilibrium, All Firms Sell Equity

We now consider the alternative pooling equilibrium in which both types issue equity, supported by the OEPB that an asset selleris of quality L. Equity is valued at

$$E[C+A] + F = \pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F$$

and if assets are issued (off the equilibrium path), they are valued at A_L .

The fundamental values of H and L are, respectively:

$$C_{H} + A_{H} - F\left(\frac{(1-\pi)(C_{H} - C_{L} + A_{H} - A_{L})}{E[C+A] + F}\right),\$$
$$C_{L} + A_{L} + F\left(\frac{\pi(C_{H} - C_{L} + A_{H} - A_{L})}{E[C+A] + F}\right),\$$

As in APE, firms of quality L makes a capital gain; however, he may still have incentives to deviate and sell assets if they are sufficiently dissynergistic. Type (L, \underline{k}) has the greatest incentive to deviate and its no-deviation condition is given by:

$$F \le F^{EPE,ND,L} = \frac{E[C+A](1+\underline{k}) - (C_L + A_L)}{-\underline{k}}.$$
(12)

A necessary condition for (12) to be satisfied is that

$$\underline{k} > \frac{C_L + A_L}{E[C+A]} - 1, \tag{13}$$

i.e. the dissynergies cannot be sufficiently large to outweigh the capital gain on selling low-quality assets. Equation (13) is analogous to equation (6) in APE which also limits the size of synergies. We assume that equation (13) holds throughout the case of positive correlation, else synergy motives are so strong that they dominate information asymmetry considerations. However, while (6) in APE was both a necessary and sufficient condition, (13) is only a necessary condition. We also require F to be low for (12) to be satisfied. If F is sufficiently high, type (L, \underline{k}) only makes a small capital gain from selling low-quality equity at a pooled price, due to the certainty effect. Thus, if \underline{k} is sufficiently negative, it will deviate to asset sales in order to get rid of a dissynergistic asset. Thus, unlike in APE, satisfying L's ND condition imposes a bound on F.

Firms of quality H will not deviate if:

$$F \ge F^{EPE,ND,H} = \frac{A_L(C_H + A_H) - A_H E[C + A](1 + \underline{k})}{A_H(1 + \underline{k}) - A_L}.$$
(14)

In contrast to Section 2.2, the ND condition now imposes a *lower* bound on F. This also results from the certainty effect. If F is sufficiently high, H suffers a relatively small loss from selling equity at a pooled price due to the certainty effect, and so will not deviate. Note that if $\frac{A_H}{A_L} > \frac{C_H + A_H}{E[C+A]}$ and $\underline{k} = 0$, so assets are sufficiently volatile and there are no synergy motives, the RHS of (14) is negative. Thus, regardless of F, H will not deviate. In contrast, recall that even if assets are less volatile than equity and there are no synergies, this is not sufficient for the ND condition in APE to be satisfied, as we also required F to be low. Low equity volatility and low dissynergies is sufficient to rule out deviation from EPE, but low asset volatility and low synergies is not sufficient to rule out deviation from APE.

We next verify whether the OEPB, that an asset seller is of quality L, satisfies the IC. If L sells assets and is inferred as quality H, his fundamental value becomes:

$$C_L + A_L + F - F\left(\frac{A_L\left(1+\underline{k}\right)}{A_H}\right).$$

Type (L, \underline{k}) thus has greatest incentive to deviate. It will do so, satisfying the IC, if

$$F \ge F^{EPE,IC} = \frac{A_L E[C+A](1+\underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1+\underline{k})}.$$
(15)

The denominator is always positive, and so the lower bound can always be satisfied for some F: unlike in APE, there is no necessary condition that we require for the IC condition to be achievable. The condition is a lower bound, since F must be sufficiently high that the certainty effect reduces the capital gain from pooling on equity, so that L prefers to deviate and sell assets.

Lemma 2 below summarizes the equilibrium. The proof shows that the IC condition is stronger than the ND condition if and only if $1 + \underline{k} \geq \frac{A_H A_L}{E[A^2]}$ (< 1). Thus, if this inequality is satisfied, the former is necessary and sufficient for the pooling equilibrium to hold, else the latter is the necessary and sufficient condition.

Lemma 2. (Positive correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium in which all firms sell equity $(K_q = E)$ and a firm that sells assets is inferred as type (L, k). This equilibrium is sustainable if the following conditions are satisfied:

(i) $F \leq F^{EPE,ND,L} = \frac{E[C+A](1+\underline{k})-(C_L+A_L)}{-\underline{k}}$ (ii) $F \geq F^{EPE}$, where

$$F^{EPE} = \begin{cases} F^{EPE,IC} = \frac{A_L E[C+A](1+\underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1+\underline{k})} & \text{if } 1 + \underline{k} \ge \frac{A_H A_L}{E[A^2]} \\ F^{EPE,ND,H} = \frac{A_L(C_H + A_H) - A_H E[C+A](1+\underline{k})}{A_H(1+\underline{k}) - A_L} & \text{if } 1 + \underline{k} < \frac{A_H A_L}{E[A^2]}. \end{cases}$$
(16)

The lower bound $F^{EPE,IC}$ is increasing in \underline{k} ; the lower bound $F^{EPE,ND,H}$ and the upper bound $F^{EPE,ND,L}$ are both decreasing in \underline{k} . The comparative statics with respect to the other parameters are ambiguous.

2.3 Comparing the Pooling Equilibria

We now study the conditions under which each pooling equilibrium is sustainable. The results are given in Proposition 1 below:

Proposition 1. (Positive correlation, comparison of pooling equilibria.) An assetpooling equilibrium is sustainable if $F \leq F^{APE}$, and an equity-pooling equilibrium is sustainable if $F^{EPE} \leq F \leq F^{EPE,ND,L}$, where F^{APE} , $F^{EPE,ND,L}$, and $F^{EPE,IC}$ are given by (11), (13) and (16) respectively and $F^{APE} > F^{EPE}$. Thus, if:

(i) $F \leq F^{EPE}$, only an asset-pooling equilibrium is sustainable,

(ii) $F^{EPE} < F \leq F^{APE}$, both the asset-pooling and equity-pooling equilibria are sustainable,

(iii) $F^{APE} < F \leq F^{EPE,ND,L}$, only an equity-pooling equilibrium is sustainable.

(iv) $F > F^{EPE,ND,L}$, no pooling equilibrium is sustainable.

Proposition 1 shows that, when the amount of financing required increases (but remains below $F^{EPE,ND,L}$), firms switch from selling assets to issuing equity, since the certainty effect becomes stronger. Thus, the type of claim issued depends not only on the inherent characteristics of the claim (its information asymmetry and synergies) but also the amount of financing required – firms only issue equity to raise large amounts of financing, even though we have no fixed costs of equity issuance. In standard theories, the type of security issued only depends on the security's inherent characteristics (information asymmetry or overvaluation) unless one assumes nonlinearities such as limited debt capacity. Here, there are no limits as the amount of financing required can be fully raised by either source.

When the financing needs become too high $(F > F^{EPE,ND,L})$, no pooling equilibrium is sustainable. Due to the certainty effect, information asymmetry considerations become second-order, and so firms with synergies (dissynergies) will sell equity (assets).

2.4 Separating Equilibrium

In a separating equilibrium, the financing choice depends on the synergy parameter k: there is a cutoff k_q^* so any firm below (above) the cutoff will sell assets (equity). In general, H and L can use different cutoff rules, so separation will be along both type dimensions.

While investors do not directly care about k (as it only affects private values), the synergy cutoffs do matter since they affect the expected quality (common value) of the claims. Thus, investors will try to infer q from the seller's choice of claim. Using Bayes' rule, the prices paid for sold assets and issued equity are, respectively:

$$E[A|k < k_q^*] = \pi \frac{k_H^* - \underline{k}}{E[k_q^*] - \underline{k}} A_H + (1 - \pi) \frac{k_L^* - \underline{k}}{E[k_q^*] - \underline{k}} A_L$$
(17)

$$E[C+A|k > k_q^*] + F = \pi \left(\frac{\overline{k} - k_H^*}{\overline{k} - E[k_q^*]}\right) (C_H + A_H) + (1 - \pi) \left(\frac{\overline{k} - k_L^*}{\overline{k} - E[k_q^*]]}\right) (C_L + A_L) + F$$
(18)

where

$$E\left[k_{q}^{*}\right] = \pi k_{H}^{*} + \left(1 - \pi\right) k_{L}^{*}$$

The effect of the different cutoff thresholds is to skew the valuations. In an APE, assets are valued at $\pi A_H + (1 - \pi) A_L$. If $k_H^* > k_L^*$, *H*-firms are more willing to sell assets than *L*-firms. Thus, asset sales are a positive signal of quality, and so the

posterior weight placed on A_H , $\pi \frac{k_H^* - \underline{k}}{E[k_q^*] - \underline{k}}$, is greater than the prior probability π .

A type (q, k) will prefer equity issuance if and only if its unit cost of financing is less:

$$\frac{C_q + A_q + F}{E[C + A|k > k_q^*] + F} < \frac{A_q(1+k)}{E[A|k < k_q^*]}$$

where the denominators are given by equations (17) and (18), respectively. This can be rewritten

$$(1+k) > \frac{C_q + A_q + F}{A_q} \frac{E\left[C + A|k > k_q^*\right] + F}{E\left[A|k < k_q^*\right]}.$$
(19)

The cutoff k_q^* for a particular quality q is that which allows (19) to hold with equality. While we cannot solve for k_q^* in closed form, we can study whether $k_H^* \leq k_L^*$. Since only the $\frac{C_q + A_q + F}{A_q}$ term on the RHS depends on q (the second term is an expectation), the higher cutoff value k_q^* will belong to the quality q for which this term is higher. Thus, $k_H^* > k_L^*$ if and only if $\frac{C_H + A_H + F}{A_H} > \frac{C_L + A_L + F}{A_L}$, i.e.

$$\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L}.$$
(20)

Condition (20) is intuitive. It requires that the "certainty-effect" adjusted volatility is higher for equity than asset sales. H dislikes volatility as it increases the capital loss that he suffers; conversely, L likes volatility. Thus, if equity is riskier, it becomes less (more) attractive to H(L); therefore, the threshold synergy below which H is willing to sell assets is higher.

While the synergy effect alone is unsurprising (firms are more likely to sell assets if they are dissynergistic, so the equilibrium is given by a cutoff rule k_q^*), of greater interest is how the certainty effect interacts with the synergy effect. The amount of financing required F changes the cutoffs and thus the quality of assets and equity sold in equilibrium, in turn affecting their prices. If F rises so that (20) is violated, the certainty effect is sufficiently strong that equity issuance is more attractive to H and less attractive to L. Thus, more H firms sell equity, increasing the quality (and thus price) of equity issued and decreasing the quality and price of assets sold.

The results of this subsection are summarized in Proposition 2 below. Since F does not affect the sustainability of the separating equilibrium¹³, but only the cutoffs, we do not include SE in the comparison of equilibria in Section 2.3.

Proposition 2. (Positive correlation, separating equilibrium.) A separating equilib-

¹³In a later draft we will provide the formal proof that SE is sustainable for all F.

rium is sustainable in which quality q sells assets if $k \leq k_q^*$ and sells equity if $k > k_q^*$.

(i) If $\frac{C_H + A_H + F}{C_L + A_L + F} < \frac{A_H}{A_L}$, $k_L^* > k_H^* > 0$. (ii) If $\frac{C_H + A_H + F}{C_L + A_L + F} = \frac{A_H}{A_L}$, $k_L^* = k_H^*$ and the signs of k_L^* and k_H^* depend on parameter values.

(iii) If $\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L}$, $k_L^* < k_H^*$ and the signs of k_L^* and k_H^* depend on parameter values.

3 **Negative Correlation**

We now turn to the case of negative correlation. Since $A_L > A_H$, we now use the term "high (low)-quality assets" to refer to the assets of type L(H). Note that negative correlation is a rather mild condition: it only means that high-quality firms are not universally high-quality, as they may have some low-quality divisions; similarly, lowquality firms may have some high-quality assets. It does not require the values of the divisions covary in opposite directions to each other (e.g. that a market upswing helps one division and hurts the other).

Under the case of negative correlation, there is a trade-off involved in selling assets: being inferred as type H maximizes the market value, but being inferred as type Lmaximizes proceeds and thus fundamental value. Since the two effects work in opposite directions, we now return to general ω , so that the manager cares about the stock price as well as fundamental value.

Throughout the negative correlation analysis, we make the following technical assumption:

$$1 + \underline{k} > \frac{A_H}{A_L}.$$
(21)

Analogous to (6) and (13) in the positive correlation case, this assumption ensures that synergies are not sufficiently strong that they swamp information asymmetry considerations. As with Section 2, we first study the pooling equilibria and then the separating equilibrium.

Pooling Equilibrium, All Firms Sell Assets 3.1

As in Section 2.1, we consider a pooling equilibrium in which all firms sell assets, supported by the OEPB that anyone who sells equity is of type (L, k). As before, sold assets are valued at $E[A] = \pi A_H + (1-\pi)A_L$ and issued equity is valued at $C_L + A_L + F$.

A firm that sells assets has a stock price of E[C + A], and an equity issuer is priced at $C_L + A_L$.

By deviating, an *L*-firm avoids the capital loss from selling highly-valued assets at a pooled price as well as any loss of synergies, but suffers a low stock price. Thus, he will only cooperate if its concern for the stock price is sufficiently high. Since (L, \overline{k}) is most likely to deviate, all *L*-firms will cooperate if:

$$\omega \ge \omega^{APE,ND,L} = \frac{F\left(\frac{A_L}{E[A]}(1+\bar{k})-1\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\frac{A_L}{E[A]}(1+\bar{k})-1\right)}.$$
 (22)

If (22) is satisfied, it is automatic that all *H*-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by pooling on asset sales. Thus, (22) is necessary and sufficient for all firms not to deviate.

The lower bound given by (22) is relatively loose, i.e., easy to satisfy. It is relatively easy to rule out a deviation to equity issuance. Issuing equity not only leads to a low price (of $C_L + A_L$) on the equity being sold (as in MM), but also implies a low valuation (of $C_L + A_L$) for the rest of the firm. This is because the correlation between the equity being sold and the rest of the firm is necessarily 1. The second effect is absent in MM, since the manager only cares about fundamental value and not the stock price. As we will see in EPE, this will not be the case when considering deviations to asset sales, since the asset being sold is not a carbon copy of the rest of the firm.

It is automatic that the OEPB satisfies the IC. Type (L, \overline{k}) will indeed deviate to equity if revealed H: his stock price will rise, he will receive a capital gain by selling equity for a high price (compared to his current loss for selling high-quality assets at a pooled price) and he avoids the loss of synergies $\overline{k} \geq 0$.

The results of this subsection are summarized in Lemma 3 below:

Lemma 3. (Negative correlation, pooling equilibrium, all firms sell assets.) A pooling equilibrium is sustainable in which both types sell assets $(K_H = K_L = A)$ and a firm that sells equity is inferred as type (L, \overline{k}) , if

$$\omega \ge \omega^{APE,ND,L} = \frac{F\left(\frac{A_L}{E[A]}(1+\overline{k})-1\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\frac{A_L}{E[A]}(1+\overline{k})-1\right)}$$

The lower bound $\omega^{APE,ND,L}$ is increasing in F, π , \overline{k} , and $A_L - A_H$, and decreasing in $C_H - C_L$.

The bound is increasing in F, so again the choice of financing depends on the amount of financing required. However, F plays a different role here than in the positive correlation model, where it was important due to the certainty effect. Here, a greater F means that L's capital loss from pooling is sustained over a larger base. It increases the fundamental value motive relative to the market value motive, and requires a higher weight on the market value ω to maintain indifference. This "base effect" was absent from the positive correlation section, as there was no trade-off between stock price and fundamental value. Put differently, if F is high, L suffers such a large capital loss from selling assets that it prefers to "bite the bullet" and issue equity even though this leads to a low market valuation. The bound is also increasing in \overline{k} : higher \overline{k} increases the fundamental loss that (L, \overline{k}) suffers from selling assets, and so we require a lower weight on fundamental value (a higher ω) for him not to deviate. The intuition behind the other comparative statics is given in Appendix B.

3.2 Pooling Equilibrium, All Firms Sell Equity

We next consider a pooling equilibrium in which all firms sell equity, supported by the OEPB that anyone who sells assets is of type (L, \underline{k}) .¹⁴ As before, issued equity is valued at E[C + A] + F and sold assets are valued at A_L . The stock price is E[C + A]for an equity issuer and $C_L + A_L - F\underline{k}$ for an asset seller.

By deviating, an *H*-firm avoids the capital loss from equity issuance and gets rid of a dissynergistic asset, but suffers a low stock price from being inferred as *L*. Since (H, \underline{k}) is most likely to deviate, all *H*-firms will cooperate if:

$$\omega \ge \omega^{EPE,ND,H} = \frac{F\left(\frac{C_H + A_H + F}{E[C+A] + F} - \frac{A_H(1+\underline{k})}{A_L}\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\underline{k} + \frac{C_H + A_H + F}{E[C+A] + F} - \frac{A_H(1+\underline{k})}{A_L}\right)}.$$
 (23)

¹⁴For all equilibria, we specify the OEPB that anyone who deviates is of quality L, and are free to choose whichever synergy parameter makes the equilibrium most likely to hold. In all equilibria considered thus far, the synergy parameter only affected the IC condition and so the choice was straightforward: we choose the synergy parameter which makes the IC condition easier to satisfy. Here, the synergy parameter affects both IC and ND and so the choice is not straightforward. A lower k makes IC easier to satisfy (as (L, \underline{k}) is more willing to deviate to asset sales to get rid of a dissynergistic asset) but increases the stock price of a deviator (as it is deemed to be losing a dissynergistic asset) and makes ND harder to satisfy. We follow the earlier equilibria and choose the k that makes the IC easiest to satisfy. This is because the goal of this section is to show that APE is sustainable for a greater range of parameters than EPE. In Section 3.3 we show that the IC condition for EPE is tighter than the ND condition for APE, so if we chose a different k (which would make the IC condition for EPE harder to satisfy), this would still hold.

If (23) is satisfied, it is automatic that all *L*-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by pooling on equity issuance. Thus, (23) is necessary and sufficient for all firms not to deviate. Comparing (23) with (22), the necessary and sufficient condition for no deviation in *APE*, we can see that the *EPE* condition is relatively more difficult to satisfy. In the *APE*, deviation to equity issuance led to a low price of $C_L + A_L$ not only on the equity being sold, but also on the rest of the firm. Here, deviation to asset sales leads to a low price of $C_L + A_L - F\underline{k}$ on the rest of the firm, but a high price of A_L on the asset being sold, since it is not a carbon copy. This difference is due to the correlation effect.

Unlike in Section 3.1, it is not automatic that the OEPB satisfies the IC. If (L, \underline{k}) deviates to asset sales and is revealed (H, \underline{k}) (the type that leads to the highest stock price), his payoff becomes

$$\omega \left(C_H + A_H - F\underline{k} \right) + \left(1 - \omega \right) \left(C_L + A_L + F - F \left(\frac{A_L(1+k)}{A_H} \right) \right)$$

He enjoys a stock price increases and loses potentially dissynergistic assets, but suffers a capital loss on asset sales. Thus, the IC condition is only satisfied if:

$$\omega \ge \omega^{EPE,IC} = \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{C_L + A_L + F}{E[C+A] + F}\right)}{(1-\pi)(C_H - C_L - (A_L - A_H)) + F\left(\underline{k} + \frac{A_L(1+\underline{k})}{A_H} - \frac{C_L + A_L + F}{E[C+A] + F}\right)}.$$
 (24)

This is also a lower bound, which is strictly between 0 and 1. From (21), the first term in the numerator exceeds 1: L suffers a fundamental loss from asset sales, because the capital loss from selling good assets at a low price is less than the gain from getting rid of a synergistic asset. If L deviates to asset sales and is inferred as H, he suffers a fundamental loss because he sells high-quality assets for a low price. Thus, only if his stock price concerns are sufficiently high will he deviate. Note that the IC was trivially satisfied in APE where the deviation involved issuing equity – if the deviator is inferred as H, he receives both a high stock price and a high value for the equity being sold: both are valued at $C_H + A_H$ since the former is a carbon copy of the latter. Here, the deviation is to assets, which are not a carbon copy of the firm and so can be priced differently: even though the deviator enjoys a high market valuation (of $C_H + A_H - F\underline{k}$), he suffers a loss on the assets being sold (which fetch only A_H). The IC condition (24) is stronger than the ND condition (23) if and only if:

$$F\underline{k} < \left[(C_H - C_L) - (A_L - A_L) \right] \frac{\left[\pi A_L^2 + (1 - \pi) A_H^2 \right] (1 + \underline{k}) - A_H A_L}{\left[A_L^2 + A_H^2 \right] (1 + \underline{k}) + 2A_H A_L}$$
(25)

Lemma 4 below summarizes the equilibrium.

Lemma 4. (Negative correlation, pooling equilibrium, all firms sell equity.) A pooling equilibrium is sustainable in which all firms sell assets ($K_H = K_L = A$) and a firm that sells assets is inferred as type L, if $\omega \geq \omega^{EPE}$, where

$$\omega^{EPE} = \begin{cases} \omega^{EPE,IC} = \frac{F\left(\frac{A_L(1+k)}{A_H} - \frac{C_L + A_L + F}{E[C+A] + F}\right)}{(1-\pi)(C_H - C_L - (A_L - A_H)) + F\left(\frac{A_L(1+k)}{A_H} - \frac{C_L + A_L + F}{E[C+A] + F}\right)} & \text{if (25) holds} \\ \omega^{EPE,ND,H} = \frac{F\left(\frac{C_H + A_H + F}{E[C+A] + F} - \frac{A_H(1+k)}{A_L}\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\frac{k}{E} + \frac{C_H + A_H + F}{E[C+A] + F} - \frac{A_H(1+k)}{A_L}\right)} & \text{if (25) does not hold.} \end{cases}$$

$$(26)$$

3.3 Comparing the Pooling Equilibria

We now study the conditions under which each pooling equilibrium is sustainable. The results are given in Proposition 1 below:

Proposition 3. (Negative correlation, comparison of pooling equilibria.) An assetpooling equilibrium is sustainable if $\omega \geq \omega^{APE,ND,L}$, and an equity-pooling equilibrium is sustainable if $\omega \geq \omega^{EPE}$, where $\omega^{APE,ND,L}$ and ω^{EPE} are given by (22) and (26), respectively and $\omega^{APE,ND,L} < \omega^{EPE}$. Thus, if:

(i) $0 < \omega < \omega^{APE,ND,L}$, neither pooling equilibrium is sustainable,

(ii) $\omega^{APE,ND,L} \leq \omega \leq \omega^{EPE}$, only the asset-pooling equilibrium is sustainable,

(iv) $\omega^{EPE} \leq \omega < 1$, both the asset-pooling and equity-pooling equilibria are sustainable.

The thresholds $\omega^{APE,ND,L}$ and ω^{EPE} are both increasing in F.

Proposition 3 shows that, for the case of negative correlation, asset sales will be more commonly used than equity issuance. The range of ω 's over which EPE is sustainable is a strict subset of the range of ω 's over which APE is sustainable. Note that the preference for asset sales exists even though assets may be riskier than core equity. If $\frac{A_L}{A_H} > \frac{C_H + A_H}{C_L + A_L}$ (which is fully consistent with the conditions $C_H + A_H > C_L + A_L$ and $A_L > A_H$), then assets are more volatile and the MM principle would suggest that equity issuance should be preferred. In contrast, we show that asset sales are may be preferred due to the correlation effect; the intuition has been described earlier. Note that this correlation effect is absent in a standard financing model of security issuance, because both debt and equity are positively correlated with the value of the firm. Thus, the issuance of debt may imply that debt is low-quality, and thus the remainder of the firm is also low-quality.

3.4 Separating Equilibrium

As in Section 2.4, we have a separating equilibrium characterized by a cutoff k_q^* . As before, the prices paid for sold assets and issued equity are given by (17) and (18). Since the manager now places weight on the firm's stock price, we need to consider the stock price inferences of the different financing choices. If a firm sells assets, its stock price becomes

$$E[C+A|k < k_q^*] = \pi \left(\frac{k_H^* - \underline{k}}{E[k_q^*] - \underline{k}}\right) (C_H + A_H) + (1-\pi) \frac{k_L^* - \underline{k}}{E[k_q^*] - \underline{k}} (C_L + A_L) \quad (27)$$

and if it issues equity, its stock price is

$$E[C+A|k>k_q^*] = \pi \left(\frac{\overline{k}-k_H^*}{\overline{k}-E[k_q^*]}\right) (C_H+A_H) + (1-\pi) \left(\frac{\overline{k}-k_L^*}{\overline{k}-E[k_q^*]}\right) (C_L+A_L).$$
(28)

Thus, type (q, k) issues equity if and only if

$$\omega \left(E[C+A|k < k_q^*] - E[C+A|k > k_q^*] \right) < (1-\omega)F\left(\frac{A_q(1+k)}{E[A|k < k_q^*]} - \frac{C_q + A_q + F}{E[C+A|k > k_q^*] + F}\right)$$
(29)

The cutoff k_q^* for a particular quality q is that which allows (29) to hold with equality. Only the parenthetical term on the RHS differs according by quality k. Ignoring k, the parenthetical term will be higher for L, since $A_L > A_H$ and $C_L + A_L < C_H + A_H$, and so $k_H^* > k_L^*$. This is intuitive: since H has lower-quality assets but higher-quality equity, it is more willing to sell assets. In Section 2.4 under positive correlation, $k_H^* > k_L^*$ only if asset are safer than equity (adjusted for the certainty effect), as this means means that the capital loss from asset sales is less than from equity issuance. With negative correlation, the capital loss from asset sales is always less since it is negative (i.e., a capital gain), so we always have $k_H^* > k_L^*$. From (27) and (28), $k_H^* > k_L^*$ implies that asset sales lead to a positive stock price reaction and equity issuance leads to a negative stock price reaction. Indeed, Jain (1985), Klein (1986), Hite, Owers, and Rogers (1987), and Slovin, Sushka, and Ferraro (1995), among others, find evidence of the former; a long line of empirical research including Asquith and Mullins (1986) documents the latter.

The amount of financing required F has two effects on the cutoffs. To illustrate, consider L's decision. On the one hand, an increase in F augments the certainty effect and makes equity issuance less attractive, because L enjoys a smaller capital gain. This tends to make L prefer asset sales and increase k_L^* . On the other hand, an increase in F augments fundamental value considerations as they are now off a higher base. Thus, the stock price boost from asset sales is relatively less important, which tends to decrease k_L^* , so the overall effect is ambiguous. Thus, the separating equilibrium of this section combines both effects of F: the certainty effect which is present in the APE under positive correlation, and the base effect which is present in the EPE under negative correlation.

A second contrast with the positive correlation case is that it is possible to have separation purely by quality and not by synergy, i.e. $k_H^* = \overline{k}$ and $k_L^* = \underline{k}$, where all high-quality firms sell assets and all low-quality firms sell equity. We use SE^q to denote a separating equilibrium by quality only. In the positive correlation case, SE^q is unsustainable as (L, \underline{k}) will deviate to sell assets and enjoy a capital gain plus a loss of dissynergies. Here, SE^q may be sustainable as, even though (L, \underline{k}) will get rid of a dissynergistic asset and enjoy a higher stock price, he will suffer a capital loss. Indeed, SE^q is sustainable if both of the following conditions are satisfied:

$$\omega \ge \omega^{SE^q,H} = \frac{F\left((1+\overline{k}) - \frac{C_H + A_H + F}{C_L + A_L + F}\right)}{\left(C_H - C_L - (A_L - A_H)\right) + F\left((1+\overline{k}) - \frac{C_H + A_H + F}{C_L + A_L + F}\right)}$$
(30)

$$\omega < \omega^{SE^{q},L} = \frac{F\left(\frac{A_{L}(1+\underline{k})}{A_{H}} - 1\right)}{(C_{H} - C_{L} - (A_{L} - A_{H})) + F\left(\frac{A_{L}(1+\underline{k})}{A_{H}} - 1\right)}.$$
(31)

The lower bound on ω ensures that H will not deviate. Type (H, \overline{k}) may wish to deviate as it is selling a synergistic asset; a high ω is needed for the stock price motive to be strong enough to deter such a deviation. If $1 + \overline{k} < \frac{C_H + A_H + F}{C_L + A_L + F}$, i.e. synergies are less important than information asymmetry, then the loss of synergies exceeds the capital loss that (H, \overline{k}) would suffer by issuing equity. Thus, H's fundamental value and stock price are both higher under asset sales, and the lower bound on ω is trivially satisfied. There are two effects on F on the lower bound – it tightens it due to the

certainty effect but also loosens it due to the base effect – so the overall impact is ambiguous.

The upper bound ensures that L will not deviate. By deviating, type (L, \underline{k}) suffers a fundamental loss: from (21), the capital loss from selling high-quality assets exceeds the benefits of getting rid of a synergistic asset. Deviating also leads to a stock price increase. Thus, ω must be low so that the stock price motive is sufficiently weak that L does not deviate. Unlike the lower bound, the upper bound on ω is unambiguously increasing in F as only the base effect exists.

The range of ω 's that satisfy (30) and (31) is increasing in \underline{k} and decreasing in k: the weaker the synergy motive, the easier it is to sustain SE^q . Indeed, if $\overline{k} = 0$, it is automatic that H will never deviate as it will suffer a stock price decline and a capital loss on equity issuance, and so the lower bound is always satisfied. If ω falls outside the bounds given by (30) and (31), a separating equilibrium may be sustainable but will involve $k_H^* < \overline{k}$ and/or $k_L^* > \underline{k}$ (rather than SE^q).

In SE^q , assets are sold for the lowest possible price of A_H and equity is issued at the lowest possible price of $C_L + A_L$, so there are no capital gains or losses. Type H's assets are correctly assessed by the market as being a "lemon", and so the market timing motive for financing (e.g. Baker and Wurgler (2002)) does not exist – in contrast to the other separating equilibria where the H-firms that sell assets and the L-firms that sell equity enjoy capital gains. However, under negative correlation, the low valuation on the assets sold does not imply a low valuation for the rest of the firm. Thus, His willing to sell assets despite receiving a low price for them. The sustainability of SE^q (which was not possible in Section 2.4) stems from the correlation effect. Type H can separate himself by choosing a claim that is not a carbon copy of (is negatively correlated with) the rest of the firm.

Thus, the correlation effect – and its implications for the desirability of financing through asset sales – manifests in two ways. First, as shown in Section 3.3, the *APE* is sustainable over a greater range of parameters than *EPE*. Second, SE^q is sustainable, unlike in the positive-correlation model. The separating equilibrium is also featured in Nanda and Narayanan (1999), who consider a model in which core and non-core assets are always negatively correlated¹⁵, and $\omega = 0$. Thus, no pooling equilibria are sustainable in the absence of transactions costs. They assume that the transactions costs of asset sales are higher than for equity issuance, which sometimes supports an *EPE* but never an *APE*, which is the opposite result to our paper.

¹⁵If these assets are positively correlated, there is no information asymmetry in their model.

This analysis points to an interesting benefit of diversification. Stein (1997) noted that an advantage of holding assets that are not perfectly correlated is that a conglomerate can engage in "winner-picking", i.e. increase investment in the division that has the best investment opportunities at the time. Our model suggests that an advantage of diversification is "loser-picking": a firm can sell a low-quality asset, thus raising financing, without implying a low value for the rest of the firm. Non-core assets may thus be seen as a form of financial slack, or financing capacity: they can be sold (even if they turn out to be poorly-performing) without adversely affecting the value of the rest of the firm. Moreover, they may even be preferable to debt capacity: as explained earlier, the value of debt is typically positively correlated with firm value, so the issuance of debt may lead the market to infer that both the debt being issued and the remainder of the firm are low-quality.

A third contrast with the positive correlation case is that, in the presence of synergies, the *H*-firms that sell assets enjoy a capital gain from doing so, since $(17) > A_H$. (Without synergies, the only separating equilibrium is SE^q where there are no capital gains or losses.) By selling assets, *H* pools with some *L* firms which have high-quality assets but choose to sell assets because they are synergistic. Put differently, *H* is able to disguise asset sales motivated by overvaluation reasons (low common value) as being instead motivated by business reasons (low private value but high value to a buyer). Thus, it does not suffer the full lemons discount and receives a price greater than A_H . As in the positive correlation case, *L*-firms that sell equity earn a capital gain as they pool with *H*-firms who choose to sell equity, even though it is high quality, as they do not wish to sell a synergistic asset.

The results of this section are summarized in Proposition 4 below.

Proposition 4. (Negative correlation, separating equilibrium.) A separating equilibrium is sustainable in which quality q sells assets if $k \leq k_q^*$ and sells equity if $k > k_q^*$, where $k_H^* > k_L^*$ and $k_H^* > 0$; the sign of k_L^* depends on parameter values. A separating equilibrium is sustainable in which all firms of quality H (L) sell assets (equity) is sustainable if the following two conditions are satisfied:

$$\omega \ge \omega^{SE,H} = \frac{F\left(\left(1+\overline{k}\right) - \frac{C_H + A_H + F}{C_L + A_L + F}\right)}{\left(C_H - C_L - \left(A_L - A_H\right)\right) + F\left(\left(1+\overline{k}\right) - \frac{C_H + A_H + F}{C_L + A_L + F}\right)}$$
$$\omega < \omega^{SE,L} = \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}{\left(C_H - C_L - \left(A_L - A_H\right)\right) + F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}.$$

4 Cash Used For Investment

This section extends the model to allow for the cash raised (F) to be used to finance an investment with an uncertain value. To make the effects of investment as clear as possible, we will focus on the no-synergies case of $\overline{k} = \underline{k} = 0$.

Since all agents are risk-neutral, only expected values matter. Thus, the model is unchanged if we simply make the investment opportunity uncertain, so that its payoff is a random variable with expected value equal to F regardless of firm quality. For the investment opportunity to affect the analysis, it must vary with firm quality so that there is information asymmetry between the manager and investors regarding its value. We thus assume that F is used to finance an investment with expected value R_q in firm type q. We parameterize $R_q = F(1 + r_q)$, where $r_H \ge 0$ and $r_L \ge 0$: since there are no agency problems in the model, the investment will only be undertaken if it is positive-NPV (this is also the case in MM). We allow for both $r_H \ge r_L$ and $r_H < r_L$. The former is likely more common in reality as high-quality firms typically have superior investment opportunities, but we also allow for $r_H < r_L$ as it may be that a firm that is currently performing poorly has greater room for improvement. In Appendix E we allow for the cases of $r_H < 0$ and $r_L < 0$ and show that the core intuitions are unchanged.

Intuitively, it would seem that, if $r_H \ge r_L$, the volatility of the investment opportunity will exacerbate the volatility of assets in place: thus, using cash for an uncertain investment will weaken the certainty effect and make equity issuance less desirable. However, we will show that this is not necessarily the case. We consider the case of positive correlation here; the case of negative correlation is very similar to the core model and is in Appendix D.

We first consider APE. Proceeding as before, H will not deviate if the "unit cost of financing" is lower for asset sales. The analog of equation (9) is now:

$$\frac{A_H}{E[A]} \le \frac{C_H + A_H + F(1 + r_H)}{C_L + A_L + F(1 + r_L)}.$$
(32)

As is intuitive, C_q and R_q (= $F(1 + r_q)$) enter symmetrically in all expressions, since the funds raised are invested. A purchaser of equity receives a share of C, R and A, but a purchaser of assets receives only a share of A. The uncertainty of the investment thus increases the volatility of core equity. From equation (32), H will not deviate if:

$$F[A_H(1+r_L) - E[A](1+r_H)] \le E[A](C_H + A_H) - A_H(C_L + A_L).$$
(33)

As in the core model (see equation (8)), if $E[A](C_H + A_H) < A_H(C_L + A_L)$, then the ND condition can never be satisfied and APE is unsustainable. We thus focus on the case in which $E[A](C_H + A_H) \ge A_H(C_L + A_L)$. We first consider $\frac{A_H}{E[A]} > \frac{1+r_H}{1+r_L}$, i.e. investment is not too volatile. The LHS of (33) is positive, and so we again have an upper bound on F, given by:

$$F \leq \frac{E[A](C_H + A_H) - A_H(C_L + A_L)}{A_H(1 + r_L) - E[A](1 + r_H)}.$$
(34)

In the core model (equation (8)), and setting synergies to zero, the denominator is $A_H - E[A]$. If $r_L > r_H$, the denominator of (34) is greater than in the core model, and so the bound is tighter: it is harder to support APE. This is intuitive: L's superior growth options counterbalance its inferior assets in place and reduce the volatility of equity, making asset sales less desirable. One may think that the reverse intuition applies to $r_H \ge r_L$: the volatility in the investment R increases the volatility in the core asset (recall that C and R enter symmetrically), making equity issuance less attractive and APE easier to sustain. Put differently, it seems that the certainty effect should become weaker since cash is now used for uncertain investment. However, this turns out not to be the case: if $r_H \ge r_L$ but $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the denominator of (34) is higher than in the core model, making APE harder to sustain.

The reason for why the above intuition is incomplete is that using cash to finance investment has two effects. They can be best seen by the following decomposition of the investment returns:

$$R_L = F (1 + r_L)$$
$$R_H = F (1 + r_L) + F (r_H - r_L)$$

The first, intuitive effect is the $F(r_H - r_L)$ term which appears in the R_H equation only. The value of the investment is greater for the high type and so it suffers a greater capital loss from selling equity. However, there is a second effect, captured by the $F(1 + r_L)$ term which is common to both types. This increases the certainty effect: since the investment is positive-NPV, an equity investor now has a claim to a larger certain value: $F(1 + r_L)$ rather than F. Put differently, while investors do not know firm quality, they do know that the funds they provide will increase in value, regardless of quality. Due to this second effect, $r_H \ge r_L$ is not sufficient for the upper bound to relax. If $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the difference in returns is insufficient to outweigh the first effect, and it is harder to prevent H from deviating. Only if $\frac{r_H}{r_L} > \frac{A_H}{E[A]}$ does the first effect dominate, leading to an increase (loosening) of the upper bound. Finally, if $\frac{A_H}{E[A]} \leq \frac{1+r_H}{1+r_L}$, i.e. investment is highly volatile, then the LHS of (33) is non-positive and so the ND condition is always satisfied: APE is sustainable for any F.

Another way to view the intuition is as follows. Equityholders obtain a portfolio of the assets in place (C + A) and the new investment (R); the size of F determines the weighting of the new investment in this portfolio. The APE is sustainable if H's capital loss from asset sales, $\frac{A_H}{E[A]}$, is less than the weighted average capital loss on this overall portfolio. If both the assets in place and the new investment are more volatile than non-core assets, i.e. $\frac{C_H + A_H}{C_L + A_L} \geq \frac{A_H}{E[A]}$ and $\frac{1+r_H}{1+r_L} \geq \frac{A_H}{E[A]}$, then the weighted average capital loss on the portfolio is greater regardless of the weights – hence, APE holds regardless of F. Deviation is only possible if the new investment is safer than non-core assets, i.e. $\frac{1+r_H}{1+r_L} < \frac{A_H}{E[A]}$. In this case, the weight placed on the new investment must be sufficiently low for the weighted average capital loss to remain higher for the portfolio, and so deviation to be ruled out. The lower the uncertainty associated with the investment, the lower the weight on it required for the overall uncertainty to be higher than for the non-core assets. Thus, holding r_L fixed, as r_H declines towards r_L , the required F goes down and the APE is harder to sustain.

Regardless of the specific values of r_H and r_L , in all cases we require the weight on the investment to be sufficiently low. Thus, the result of the core model, that F must be low for APE to be sustainable, continues to hold when cash is used to finance an uncertain investment, and regardless of the values of r_H and r_L .

The IC condition is satisfied if:

$$F(E[A](1+r_L) - A_L(1+r_H)) \le A_L(C_H + A_H) - E[A](C_L + A_L).$$
(35)

The contrast with the core model (equation (10)) is similar as for the ND conditions. If $\frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L}$, the LHS of equation (35) is non-positive and so the IC condition is satisfied for all F. If instead $\frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L} \leq \frac{r_H}{r_L}$, the upper bound on F becomes looser than in the core model. The volatility of the investment increases L's incentives to deviate and be revealed as H, since he will receive a capital gain on the investment value R in addition to the core asset value C. However, if $\frac{E[A]}{A_L} > \frac{r_H}{r_L}$, then the bound becomes tighter. As with the ND condition, this holds if $r_L > r_H$, as is intuitive (L's superior investment opportunity counterbalances its inferior assets in place), but can also hold even if $r_H \ge r_L$: since the investment is positive-NPV, it increases the certainty effect and thus reduces the desirability of issuing equity, even if L is revealed as type H. If and only if

$$\frac{1+r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}.$$
(36)

the IC condition is stronger than the ND condition, and thus is necessary and sufficient for APE to be sustainable. Noting that $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$, the equilibrium is summarized in Lemma 5 below:

Lemma 5. (Positive correlation, pooling equilibrium, all firms sell assets, cash used for investment.) A pooling equilibrium is sustainable in which all firms sell assets $(K_H = K_L = A)$ and a firm that sells equity is inferred as type L, if

$$F[A_H(1+r_L) - E[A](1+r_H)] \le E[A](C_H + A_H) - A_H(C_L + A_L)$$
(37)

$$F(E[A](1+r_L) - A_L(1+r_H)) \le A_L(C_H + A_H) - E[A](C_L + A_L).$$
(38)

(i) If $\frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L}$, the asset-pooling equilibrium is sustainable for all F. (ii) If $\frac{E[A]}{A_L} > \frac{1+r_H}{1+r_L} \geq \frac{A_H}{E[A]}$, or $\frac{1+r_H}{1+r_L} < \frac{A_H}{E[A]}$ and (36) is satisfied, the asset-pooling equilibrium is sustainable if $F \leq \frac{A_L(C_H+A_H)-E[A](C_L+A_L)}{E[A](1+r_L)-A_L(1+r_H)}$. Compared to the case where cash remains on the balance sheet (Lemma 1):

(a) If $\frac{r_H}{r_L} < \frac{E[A]}{A_L}$, the upper bound on F is tighter and the asset-pooling equilib $rium \ is \ sustainable \ across \ a \ smaller \ range \ of \ F,$

(b) If $\frac{r_H}{r_L} \geq \frac{E[A]}{A_L}$, the upper bound on F is weakly looser and the asset-pooling equilibrium is sustainable across a larger range of F,

(iii) If $\frac{1+r_H}{1+r_L} < \frac{A_H}{E[A]}$ and (36) is not satisfied, the asset-pooling equilibrium is sustainable if $F \leq \frac{E[A](C_H+A_H)-A_H(C_L+A_L)}{A_H(1+r_L)-E[A](1+r_H)}$. Compared to Lemma 1: (a) If $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the upper bound on F is tighter and the asset-pooling equilibrium is a subset of the set of t

rium is sustainable across a smaller range of F,

(b) If $\frac{r_H}{r_L} \geq \frac{A_H}{E[A]}$, the upper bound on F is weakly looser and the asset-pooling equilibrium is sustainable across a larger range of F.

We now turn to *EPE*. The effect of using cash for uncertain investment is similar to APE. Intuitively, it may seem that this usage will always make EPE harder to satisfy because the volatility of the investment reduces the certainty effect. However, if r_H is sufficiently close to r_L , this volatility effect is outweighed by the fact that the investment is positive-NPV. Since the economics are similar, we move immediately to the statement of the equilibrium in Lemma 6 below and defer the full analysis to the proofs.

Lemma 6. (Positive correlation, pooling equilibrium, all firms sell equity, cash used for investment.) A pooling equilibrium is sustainable in which all firms sell equity $(K_H = K_L = E)$ and a firm that sells assets is inferred as type L, if

$$F[A_H(1+E[r_q]) - A_L(1+r_H)] \ge A_L(C_H + A_H) - A_H E[C+A]$$
(39)

$$F(A_H(1+r_L) - A_L(1+E[r_q])) \ge A_L E[C+A] - A_H(C_L + A_L).$$
(40)

where $E[r_{a}] = \pi r_{H} + (1 - \pi)r_{L}$.

(i) If $\frac{1+r_H}{1+E[r_q]} \ge \frac{A_H}{A_L}$, the asset-pooling equilibrium is unsustainable for all F. (ii) If $\frac{1+E[r_q]}{1+r_L} > \frac{A_H}{A_L} > \frac{1+r_H}{1+E[r_q]}$, or $\frac{1+E[r_q]}{1+r_L} < \frac{A_H}{A_L}$ and (36) is satisfied, the asset-pooling equilibrium is sustainable if $F \ge \frac{A_L E[C+A] - A_H(C_L + A_L)}{A_H(1+r_L) - A_L(1+E[r_q])}$. Compared to the case where cash remains on the balance sheet (Lemma 2):

(a) If $\frac{E[r_q]}{r_L} < \frac{A_H}{A_L}$, the lower bound on F is looser and the equity-pooling equilibrium is sustainable across a greater range of F

(b) If $\frac{E[r_q]}{r_L} \ge \frac{A_H}{A_L}$, the lower bound on F is weakly tighter and the equity-pooling equilibrium is sustainable across a smaller range of F

(iii) If $\frac{1+E[r_q]}{1+r_L} > \frac{A_H}{A_L}$ and (36) is not satisfied, the asset-pooling equilibrium is sustainable if $F \ge \frac{A_L(C_H+A_H)-A_HE[C+A]}{A_H(1+E[r_q])-A_L(1+r_H)}$. Compared to the case where cash remains on the balance sheet (Lemma 2):

(a) If $\frac{r_H}{E[r_q]} < \frac{A_H}{A_L}$, the lower bound on F is tighter and the equity-pooling equilibrium is sustainable across a smaller range of F,

(b) If $\frac{r_H}{E[r_q]} \ge \frac{A_H}{A_L}$, the lower bound on F is weakly looser and the equity-pooling equilibrium is sustainable across a larger range of F.

The comparison of equilibria is summarized in Proposition 5:

Proposition 5. (Positive correlation, cash used for investment, comparison of equilibria.) An asset-pooling equilibrium is sustainable if $F \leq F^{APE,I}$, and an equity-pooling equilibrium is sustainable if $F > F^{EPE,I}$, where $F^{APE,I}$ and $F^{EPE,I}$ are given by:

$$F^{APE,I} = \begin{cases} \frac{A_L(C_H + A_H) - E[A](C_L + A_L)}{E[A](1 + r_L) - A_L(1 + r_H)} & \text{if } \frac{E[A]}{A_L} > \frac{1 + r_H}{1 + r_L} \ge \frac{A_H}{E[A]}, \text{ or } \frac{1 + r_H}{1 + r_L} < \frac{A_H}{E[A]} \text{ and } (36) \text{ holds,} \\ \frac{E[A](C_H + A_H) - A_H(C_L + A_L)}{A_H(1 + r_L) - E[A](1 + r_H)} & \text{if } \frac{1 + r_H}{1 + r_L} < \frac{A_H}{E[A]} \text{ and } (36) \text{ does not hold,} \\ \infty & \text{if } \frac{1 + r_H}{1 + r_L} \ge \frac{E[A]}{A_L} \\ \end{cases} \\ F^{EPE,I} = \begin{cases} \frac{A_L E[C + A] - A_H(C_L + A_L)}{A_H(1 + r_L) - A_L(1 + E[r_q])} & \text{if } \frac{1 + r_H}{1 + E[r_q]} > \frac{A_H}{A_L} > \frac{1 + E[r_q]}{1 + r_L}, \text{ or } \frac{1 + r_H}{1 + E[r_q]} < \frac{A_H}{A_L} \text{ and } (36) \text{ holds,} \\ \frac{A_L(C_H + A_H) - A_H E[C + A]}{A_H(1 + E[r_q]) - A_L(1 + r_H)} & \text{if } \frac{1 + r_H}{1 + E[r_q]} > \frac{A_H}{A_L} \text{ and } (36) \text{ does not hold,} \\ \infty & \text{if } \frac{1 + E[r_q]}{A_H(1 + E[r_q]) - A_L(1 + r_H)} & \text{if } \frac{1 + r_H}{1 + E[r_q]} > \frac{A_H}{A_L} \text{ and } (36) \text{ does not hold,} \\ \infty & \text{if } \frac{1 + E[r_q]}{1 + r_L}. \end{cases}$$

The thresholds $F^{APE,I}$ and $F^{EPE,I}$ are both increasing in r_H and decreasing in r_L .

Proposition 5 demonstrates the robustness of the results of the core model to allowing cash to be used for a volatile investment rather than remaining on the balance sheet. Regardless of r_H and r_L , it remains the case that APE is sustainable for low Fand the EPE is sustainable for high F: it is never the case that the volatility of the investment causes the certainty effect to "reverse" and mean that asset (equity)-pooling is now sustainable for high (low) F. As in the core model, the source of financing depends on the amount of financing raised.

In addition to demonstrating the robustness of this idea from the core model, this extension also demonstrates a new result. As r_H rises and r_L falls, the upper bound on APE loosens and the lower bound on the EPE tightens. Indeed, if r_H is sufficiently greater than r_L , the bound becomes infinite: APE is sustainable for all F (since the upper bound is now infinity) and EPE is sustainable for no F (since the lower bound is now infinity). This can cause the equilibrium to shift from equity issuance to asset sales.¹⁶ Thus, the source of financing also depends on the use of financing: if the funds will be used for volatile investments, it is more likely to be raised from asset sales rather than equity issuance. The source of financing can depend on the use of financing in models of moral hazard (uses that are more likely to be subject to agency problems will be financed by debt rather than equity, to avoid the agency costs of dispersed equity) or bankruptcy costs (purchases of tangible assets are more likely to financed by debt rather than equity); here we deliver this dependence in a model of pure adverse selection, without moral hazard or bankruptcy costs. In addition, our predictions for the use of equity differ from a moral hazard model. With moral hazard, if cash is to remain on the balance sheet (rather than to finance a specific investment), debt will be preferred to equity to avoid the agency costs of free cash flow (Jensen (1986)). Here, equity is preferred due to the certainty effect.

5 Conclusion

This paper has studied a firm's choice between raising financing through selling noncore assets, and issuing equity. One relevant consideration is the relative volatility of non-core assets and equity value, a natural extension of the MM insight. This paper

¹⁶Formally, a given F under which both pooling equilibria were sustainable in the core model may now support only APE, when cash is used for investment. A given F under which only the EPE was sustainable in the core model may now support both equilibria, or only APE.

introduces three important additional effects that drive a firm's financing decision. First, investors in an equity issue share in the value of the cash raised from the issue, but purchasers of non-core assets do not. Since the value of cash is certain, this mitigates the information asymmetry associated with issuing uncertain claims: the certainty effect. Thus, even if the firm's equity is more volatile than its non-core assets, an equity issue may be preferred to an asset sale (in contrast to the MM prediction) if the financing need is sufficiently high. A firm's choice of financing thus depends on the level of financing required – low (high) financing needs are met through asset sales and high financing needs are met through equity (asset) sales. This result remains robust to allowing the cash to be used to finance an uncertain investment.

Second, the choice of financing mechanism may also depend on business motives (synergies). Interestingly, the synergy effect interacts with the certainty effect. Even if some firms own dissynergistic assets, an equity-pooling equilibrium is sustainable if the financing need is sufficiently high. In a separating equilibrium where a firm's level of synergy affects its financing choice, a higher financing need pushes high-quality firms towards equity issuance and reduces the quality and price of assets sold in equilibrium. The synergy motive also allows firms to enjoy a capital gain on selling low-quality assets, even in a separating equilibrium where both types of claim are issued. The firm is able to disguise its asset sale, which is in reality motivated by the asset's low quality, as instead being motivated by business reasons (dissynergies).

Third, a disadvantage of equity issuance is that the market attaches not only a low valuation to the equity being sold, but also to the remainder of the firm, since the equity being sold and the remaining equity are necessarily perfectly correlated. This need not be the case for an asset sale, since the asset being sold is not a carbon copy of the firm. Thus, even if the market correctly assesses the sold asset to be lowly valued, and so the firm suffers a "lemons" discount on the sold asset, this does not imply a low valuation for the firm as a whole. This correlation effect can lead to asset sales being strictly preferred to equity issuance.

In ongoing work, we are allowing firms to choose whether to raise financing. Financing needs are privately known; even if a firm does not need to raise financing, it can choose to do so. This extension captures an additional advantage of asset sales. Typically, the need to raise financing is a negative signal as it implies that the firm's cash position is weaker than previously thought (e.g. Miller and Rock (1985)). However, if a firm raises financing by selling assets, it can disguise a financing need as one that is business-motivated, i.e. driven by the desire to dispose of a non-synergistic asset. Since the market does not know the motive for the asset sale, the reaction to raising financing via selling assets is less negative.

An additional extension allows the firm to sell a claim to the core asset alone. This extension demonstrates the robustness of the ideas of the core model. One of the assets (core or non-core) will be more volatile than the other, and so the volatility of equity will lie in between. It may therefore seem (from MM) that the sale of one asset will always dominate equity issuance, since one of the assets will have lower volatility than equity. However, even though equity is not the safest claim, it may still be preferred due to the certainty effect – indeed, in the core model, the asset-pooling equilibrium may be unsustainable, and the equity-pooling equilibrium may be sustainable, even if equity is riskier than assets. Allowing the firm to sell the core asset in the negative-correlation model of Section 3 also gives the firm a choice of the asset is positively- or negatively-correlated. In the extension, the firm can either sell the core asset (which is positively-correlated with firm value) or the non-core asset (which is negatively-correlated). We will provide full analyses of these extensions in a future draft.

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A Proofs

Proof of Lemma 1

The IC condition (10) is stronger than the ND condition (8) if and only if

$$\frac{(C_H + A_H)A_L(1 + \bar{k}) - E[A](C_L + A_L)}{E[A] - A_L(1 + \bar{k})} < \frac{(C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - E[A]}$$

This yields:

$$A_{H}A_{L}(1+\overline{k})^{2}((C_{H}+A_{L})-(C_{L}+A_{L})< E^{2}[A]((C_{H}+A_{L})-(C_{L}+A_{L}))$$
$$A_{H}A_{L}(1+\overline{k})^{2}< E^{2}[A]$$
$$(1+\overline{k})< \frac{E[A]}{\sqrt{A_{H}A_{L}}}.$$

Proof of Lemma 2

 $F^{EPE,IC}$ is greater than $F^{EPE,ND,H}$ if and only if

$$\frac{A_L E[C+A](1+\underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1+\underline{k})} > \frac{A_L(C_H + A_H) - A_H E[C+A](1+\underline{k})}{A_H(1+\underline{k}) - A_L}$$

Cross multiplying and canceling yields:

$$A_{H}A_{L}(C_{L} + A_{L}) - A_{H}^{2}(C_{L} + A_{L})(1 + \underline{k}) - A_{L}^{2}E[C + A](1 + \underline{k})$$

> $A_{H}A_{L}(C_{H} + A_{H}) - A_{H}^{2}(E[C + A])(1 + \underline{k}) - A_{L}^{2}(C_{H} + A_{H})(1 + \underline{k})$

which becomes:

$$\pi A_{H}^{2}(C_{H} - C_{L} + A_{H} - A_{L})(1 + \underline{k}) + (1 - \pi)A_{L}^{2}(C_{H} - C_{L} + A_{H} - A_{L})$$

$$> A_{H}A_{L}(C_{H} - C_{L} + A_{H} - A_{L})$$

$$(1 + \underline{k})(\pi A_{H}^{2} + (1 - \pi)A_{L}^{2}) > A_{H}A_{L}$$

$$(1 + \underline{k}) > \frac{A_{H}A_{L}}{\pi A_{H}^{2} + (1 - \pi)A_{L}^{2}} = \frac{A_{H}A_{L}}{E[A^{2}]}.$$

Proof of Proposition 2

The core text has already proven that $k_H^* > k_L^*$ if and only if $\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L}$, so

we only need to discern the signs of the cutoffs k_q^* . From (19), $k_q^* > 0$ if and only if:

$$(C_q + F)(E[C + A|k > k_q^*] + F) + A_q(E[C|k > k_q^*] + F) > A_q(E[A|k < k_q^*] - E[A|k > k_q^*])$$

From the logic surrounding (20), if $\frac{C_L+A_L+F}{A_L} > \frac{C_H+A_H+F}{A_H}$, then $k_H^* < k_L^*$ and then $E[A|k < k_q^*] < E[A|k > k_q^*]$ (the assets of an asset seller are perceived to be less valuable than those of an equity issuer), so the RHS of the above is negative and the inequality is always satisfied. Thus we have $0 < k_H^* < k_L^*$. If $\frac{C_L+A_L+F}{A_L} < \frac{C_H+A_H+F}{A_H}$, the sign of the inequality depends on parameter values.

Proof of Lemma 4

Since $k_H^* > k_L^*$, the LHS of (29) is positive. Thus, the RHS must also be positive when evaluated at k_q^* to achieve equality. Since the RHS is increasing in k_q^* and is negative when q = H and $k_H^* = 0$, this we must have $k_H^* > 0$. This argument does not hold for k_L^* , since the RHS evaluated at q = L and $k_L^* = 0$ is already positive. If the LHS is larger than this value, we may have $k_L^* > 0$ as well. Thus, the sign of k_L^* depends on parameter values.

Proof of Lemma 5

For part (i) of the Lemma, we need to prove that $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$, so that if $\frac{1+r_H}{1+r_L} \ge \frac{E[A]}{A_L}$, we have $\frac{1+r_H}{1+r_L} > \frac{A_H}{E[A]}$ and so both (37) and (38) are satisfied for all F. $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$ is equivalent to:

$$(A_H - A_L)\pi^2 + 2A_L\pi - A_L > 0.$$

The roots of the LHS are:

$$\pi = \frac{\pm \sqrt{A_H A_L} - A_L}{A_H - A_L}$$

and only the positive root is negative. Since the quadratic in π is concave, the inequality holds when π exceeds the positive root, i.e.

$$\pi > \frac{\sqrt{A_H A_L} - A_L}{A_H - A_L}$$

Since we can show $\frac{\sqrt{A_H A_L} - A_L}{A_H - A_L} < \frac{1}{2}$ by algebraic manipulation, and $\pi > \frac{1}{2}$, we have $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$.

In addition, (38) is stronger than (37) if and only if

$$A_H A_L \left[(C_H + A_H)(1 + r_L) - (C_L + A_L)(1 + r_H) \right]$$

< $E^2 [A] \left[(C_H + A_H)(1 + r_L) - (C_L + A_L)(1 + r_H) \right]$

Since $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$ implies $A_H A_L < E^2[A]$, (38) is stronger than (37) if and only if $(C_H + A_H)(1 + r_L) > (C_L + A_L)(1 + r_H)$, i.e. (36) holds.

Proof of Lemma 6

We start with the ND condition. By pooling, type H's fundamental value is

$$C_H + A_H + R_H - F\left(\frac{C_H + A_H + R_H}{E[C + A + R]}\right).$$

By deviating, it becomes:

$$C_H + A_H + R_H - F\left(\frac{A_H}{A_L}\right).$$

Thus, he will not deviate if:

$$F[A_H(1+E[r_q]) - A_L(1+r_H)] \ge A_L(C_H + A_H) - A_H E[C+A]$$

where

$$E[r_q] = \pi r_H + (1 - \pi)r_L.$$

We now move to the IC condition. By pooling, type L's fundamental value is

$$C_L + A_L + R_L - F\left(\frac{C_L + A_L + R_L}{E[C + A + R]}\right).$$

By deviating to asset sales and being inferred as type H, it becomes:

$$C_L + A_L + R_L - F\left(\frac{A_L}{A_H}\right).$$

Thus, he will deviate if:

$$F[A_H(1+r_L) - A_L(1+E[r_q])] \ge A_L E[C+A] - A_H(C_L + A_L).$$

For part (i) of the Lemma, we need to prove that $\frac{1+E[r_q]}{1+r_L} > \frac{1+r_H}{1+E[r_q]}$, so that if

 $\frac{1+r_H}{1+E[r_q]} \geq \frac{A_H}{A_L}, \text{ we have } \frac{1+E[r_q]}{1+r_L} > \frac{A_H}{A_L} \text{ and so both (39) and (40) are violated for all } F.$ $\frac{1+E[r_q]}{1+r_L} > \frac{1+r_H}{1+E[r_q]} \text{ is equivalent to:}$

$$\pi^2(r_H - r_L) + 2\pi(1 + r_L) - (1 + r_L) > 0$$

and the only positive root is:

$$\frac{\sqrt{(1+r_H)(1+r_L)} - (1+r_L)}{(1+r_H) - (1+r_L)}.$$

Since we can show $\frac{\sqrt{(1+r_H)(1+r_L)}-(1+r_L)}{(1+r_H)-(1+r_L)} < \frac{1}{2}$ by algebraic manipulation, and $\pi > \frac{1}{2}$, we have $\frac{1+E[r_q]}{1+r_L} > \frac{1+r_H}{1+E[r_q]}$.

In addition, (40) is stronger than (39) if and only if

$$(\pi A_H^2 + (1 - \pi) A_L^2) [(C_H + A_H)(1 + r_L) - (C_L + A_L)(1 + r_H)]$$

> $A_H A_L [[(C_H + A_H)(1 + r_L) - (C_L + A_L)(1 + r_H)]]$

We start by proving that $\pi A_H^2 + (1 - \pi)A_L^2 > A_H A_L$. Algebraic manipulation, and using the fact that $A_H - A_L > 0$, yields:

$$\pi > \frac{A_L}{A_H + A_L}$$

Since we can show $\frac{A_L}{A_H+A_L} < \frac{1}{2}$ by algebraic manipulation, and $\pi > \frac{1}{2}$, we have $\pi A_H^2 + (1-\pi)A_L^2 > A_HA_L$. Thus, (40) is stronger than (39) if and only if $(C_H + A_H)(1+r_L) > (C_L + A_L)(1+r_H)$, i.e. (36) holds.

Proof of Lemma 7

As in the core model, it is automatic that L will not deviate. Following similar steps to the core model, H will not deviate if:

$$\omega \ge \frac{F\left(\frac{C_H + A_H + F(1 + r_H)}{E[C + A] + F(1 + E[r_q])} - \frac{A_H}{A_L}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{C_H + A_H + F(1 + r_H)}{E[C + A] + F(1 + E[r_q])} - \frac{A_H}{A_L}\right)}$$

and the IC condition is satisfied if:

$$\omega \ge \frac{F\left(\frac{A_L}{A_H} - \frac{C_L + A_L + F(1 + r_L)}{E[C + A] + F \times E[1 + r_q]}\right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L + A_L + F(1 + r_L)}{E[C + A] + F \times E[1 + r_q]}\right)}$$

Using similar steps to the proof of Proposition 4, the IC condition is stronger than the ND condition.