

# An Equilibrium Asset Pricing Model with Labor Market Search

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## Abstract

Search frictions in the labor market help explain the equity premium in the financial market. We embed the Diamond-Mortensen-Pissarides search framework into a dynamic stochastic general equilibrium economy. With standard parameter values, the model produces a sizeable equity premium of 3.67% per annum with a low interest rate volatility of 1.44%. The equity premium is strongly countercyclical, and predictable with labor market tightness, a pattern we also confirm in the data. Intriguingly, search frictions, combined with a small labor surplus and large job destruction flows, give rise endogenously to rare disaster risks à la Rietz (1988) and Barro (2006).

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# 1 Introduction

Modern asset pricing research has enjoyed much success in specifying preferences and cash flow dynamics necessary to explain the equity premium, its volatility, and its cyclical variation in the endowment economy (e.g., Campbell and Cochrane (1999); Bansal and Yaron (2004)). Explaining the equity premium in the production economy with endogenous cash flows has proven more challenging. The prior literature mostly treats cash flows as dividends. However, wages comprise about two thirds of aggregate disposable income in the data, and dividends represent only a small fraction. As such, an equilibrium macroeconomic model of asset pricing should take the labor market seriously.

We tackle the equity premium puzzle by incorporating search frictions in the labor market (e.g., Diamond (1982); Mortensen (1982); Pissarides (1985, 2000)) into a dynamic stochastic general equilibrium economy. With recursive Epstein and Zin (1989) preferences, a representative household pools incomes from its employed and unemployed members, and decides on optimal consumption and asset allocation. The unemployed members search for job vacancies posted by a representative firm. The rate at which a job vacancy is filled is affected by the congestion in the labor market. The degree of congestion is measured by labor market tightness, defined as the ratio of the number of job vacancies over the number of unemployed workers. Deviating from Walrasian equilibrium, search frictions create rents to be divided between the firm and the workers through the wage rate, which is in turn determined by the outcome of a generalized Nash bargaining process.

We find that search frictions are important for understanding the equity premium in the production economy. With reasonable parameter values, the search economy reproduces an equity premium of 3.67% per annum and a low interest rate volatility of 1.44%. The equity risk premium is also strongly countercyclical in the model. In particular, the vacancy-unemployment ratio forecasts stock market excess returns with a negative slope, a pattern that we confirm in the data. However, the model produces a market volatility of 7.83%, which is low, and an average interest rate of 3.75%, which is high relative to the data. Further, the model is partially successful in

replicating labor market moments. It produces a standard deviation of the vacancy-unemployment ratio of 0.16, albeit lower than 0.26 in the data, and a Beveridge curve with a negative vacancy-unemployment correlation of  $-0.51$ , albeit lower (in magnitude) than  $-0.91$  in the data.

Intriguingly, the search economy exhibits rare but deep macroeconomic disasters. In the stationary distribution from the model's simulations, the unemployment rate is positively skewed with a long right tail. The mean unemployment rate is about 12%, and the 5 percentile is nearby, 7.81%. However, the percentile is far from the mean, 21.51%. As such, output and consumption are both negatively skewed with a long left tail, giving rise to disasters emphasized in Rietz (1988) and Barro (2006). Applying Barro and Ursúa's (2008) peak-to-trough measurement on the simulated data, we find that the disasters in the model have the same average size, about 20%, as that in the data. However, the disaster probabilities and average duration are somewhat higher than those in the data.

The model's success in matching a sizeable and time-varying equity risk premium is remarkable, and contrasts sharply with the performance of standard production economies. In these economies, often with capital as the only productive input, the amount of endogenous risk is too small, giving rise to a tiny and time-invariant equity premium. In contrast, two key aspects of the search economy, when combined, are capable of producing a high and countercyclical amount of risk.

First, we calibrate the value of unemployment activities to be (relatively) high, meaning that the labor surplus (output minus wage) is small. Intuitively, a high value of unemployment activities implies that the wage rate is less elastic to labor productivity. By dampening the procyclical covariation of the wage rate, a small labor surplus magnifies the procyclical covariation (risk) of the residual payments to shareholders, thereby raising the equity risk premium. Further, the quantitative effect of the inelastic wage rate is especially important in bad times, when the labor surplus is even smaller due to lower productivity, further magnifying the risk and risk premium. As such, the equity risk premium is strongly countercyclical in the search economy.

Second, in standard economies, swings in cyclical investment flows have little impact on the

proportionally large capital stock. In contrast, job creation and destruction flows are substantially larger than investment and disinvestment flows in the data. In particular, the rate of capital depreciation is around 12% per annum, but the job destruction rate can be as high as 60% (e.g., Davis, Faberman, and Haltiwanger (2006)). When combined with search frictions, in particular the cost of posting vacancies that is high relative to the small labor surplus, such large job destruction flows create rare economic disasters. In contrast, such disaster dynamics are absent in standard economies.

Our work contributes to the disaster risks literature. Rietz (1988) introduces rare disasters into Mehra and Prescott's (1985) endowment economy, and argues that this extension helps explain the equity premium puzzle. In a stream of influential work, Barro (2006, 2009), Barro and Ursúa (2008), and Barro and Jin (2011) revive Rietz's insight by examining long-term data including many disaster events from a diverse set of countries (see also Reinhart and Rogoff (2009)). We show that search frictions shed light on the endogenous nature of disaster risks.

Gourio (2010) embeds rare disasters exogenously into a real business cycle model, and argues that disaster risks can explain large and countercyclical risk premiums. However, Gourio defines dividend as levered output and treats the claim to the levered output as equity. The return on capital, which is the stock return in production economies, has a low risk premium and a small volatility in his model. In contrast, by riding on search frictions, our economy features a sizeable equity risk premium, which is also time-varying, while using the standard definition of the stock return.

Our work also adds to aggregate asset pricing with production. Rouwenhorst (1995) shows that the standard real business cycle model fails to explain the equity premium because of consumption smoothing. With internal habit, Jermann (1998) uses capital adjustment costs, and Boldrin, Christiano, and Fisher (2001) use across-sector immobility to restrict consumption smoothing to reproduce a high equity premium. But both models struggle with excessively volatile interest rates.

Using recursive preferences to lower interest rate volatility, Kaltenbrunner and Lochstoer (2010) show that a production economy with capital adjustment costs still fails to reproduce a high equity

premium (see also Campanale, Castro, and Clementi (2010); Croce (2010)). Danthine and Donaldson (2002) and Favilukis and Lin (2011) show that staggered wage contracting is important for asset pricing. However, none of these studies examine asset pricing implications of search frictions.

Our work is built on the recent labor search literature. Shimer (2005) argues that the unemployment volatility in the standard search model is too low relative to that in the data. Hagedorn and Manovskii (2008) argue that the small labor surplus assumption goes a long way toward resolving the Shimer puzzle. Embedding the search model into an equilibrium asset pricing framework, we show that a similar economic mechanism also helps explain the equity premium puzzle.

The rest of the paper is organized as follows. Section 2 constructs the model. Section 3 presents the quantitative results under the benchmark calibration. Section 4 conducts an extensive set of comparative statics. Finally, Section 5 concludes.

## **2 The Economy**

We describe the model in Section 2.1, and calibrate it in Section 2.2. We discuss the solution methods in Section 2.3, and examine the properties of the model's solution in Section 2.4.

### **2.1 The Environment**

To study the interaction between search frictions and asset prices, we embed the standard Diamond-Mortensen-Pissarides search model into a dynamic stochastic general equilibrium economy.

#### **2.1.1 Search and Matching**

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995), we use the representative family construct, which implies perfect consumption insurance. In particular, the household has a continuum of employed workers and unemployed workers that are representative of the population at large. The household pools their incomes together before choosing per capita consumption and asset holdings.

The representative firm posts a number of job vacancies,  $V_t$ , to attract unemployed workers,  $U_t$ , at the unit cost of  $\kappa$ . Vacancies are filled via a constant returns to scale matching function,  $G(U_t, V_t)$ . We specify the matching function as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}, \quad (1)$$

in which  $\iota > 0$  is a constant parameter. This matching function, originated from Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

Specifically, define  $\theta_t \equiv V_t/U_t$  as the vacancy-unemployment ( $V/U$ ) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate),  $f(\theta_t)$ , is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{(1 + \theta_t^{-\iota})^{1/\iota}}, \quad (2)$$

and the probability for a vacancy to be filled per unit of time (the vacancy filling rate),  $q(\theta_t)$ , is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^\iota)^{1/\iota}}. \quad (3)$$

As such,  $f(\theta_t) = \theta_t q(\theta_t)$  and  $\partial q(\theta_t)/\partial \theta_t < 0$ , meaning that an increase in the relative scarcity of unemployed workers relative to job vacancies makes it more difficult for a firm to fill a vacancy. In this sense,  $\theta_t$  is a measure of labor market tightness from the perspective of the firm.

Once matched, jobs are destroyed at an exogenous and constant rate of  $s$  per period. As such, total employment,  $N_t$ , evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t. \quad (4)$$

We normalize the size of the work force to be one, meaning that  $U_t = 1 - N_t$ . As such,  $N_t$  and  $U_t$  can also be interpreted as the rates of employment and unemployment, respectively.

### 2.1.2 The Representative Firm

The firm takes aggregate labor productivity,  $X_t$ , as given. The law of motion of  $x_t \equiv \log(X_t)$  is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \quad (5)$$

in which  $0 < \rho < 1$  is the persistence parameter,  $\sigma > 0$  is the conditional volatility, and  $\epsilon_{t+1}$  is an independently and identically distributed standard normal shock.

The firm uses labor to produce with a constant returns to scale production technology,

$$Y_t = X_t N_t, \quad (6)$$

in which  $Y_t$  is output. The dividend to the firm's shareholders is given by:

$$D_t = X_t N_t - W_t N_t - \kappa V_t, \quad (7)$$

in which  $W_t$  is the wage rate (to be determined later in Section 2.1.4).

Let  $M_{t+\Delta t}$  be the representative household's stochastic discount factor from time  $t$  to  $t + \Delta t$ . Taking the matching probability,  $q(\theta_t)$ , and the wage rate,  $W_t$ , as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted  $S_t$ :

$$S_t = \max_{\{V_{t+\Delta t}, N_{t+\Delta t+1}\}_{\Delta t=0}^{\infty}} E_t \left[ \sum_{\Delta t=0}^{\infty} M_{t+\Delta t} (X_{t+\Delta t} N_{t+\Delta t} - W_{t+\Delta t} N_{t+\Delta t} - \kappa V_{t+\Delta t}) \right], \quad (8)$$

subject to equation (4) and a nonnegativity constraint on job vacancies:

$$V_t \geq 0. \quad (9)$$

Because  $q(\theta_t) > 0$ , this constraint is equivalent to  $q(\theta_t)V_t \geq 0$ . As such, the only source of job destruction in the model is the exogenous separation between the employed workers and the firm.

Let  $\mu_t$  denote the Lagrange multiplier on the employment accumulation equation (4), and  $\lambda_t$  the multiplier on the nonnegativity constraint  $q(\theta_t)V_t \geq 0$ . The first-order conditions with respect

to  $V_t$  and  $N_{t+1}$  in maximizing the equity value of equity are given by, respectively:

$$\mu_t = \frac{\kappa}{q(\theta_t)} - \lambda_t, \quad (10)$$

$$\mu_t = E_t [M_{t+1} [X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}]]. \quad (11)$$

Combining the two first-order conditions yields the intertemporal job creation condition:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t \left[ M_{t+1} \left[ X_{t+1} - W_{t+1} + (1-s) \left( \frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right] \right]. \quad (12)$$

The optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \lambda_t \geq 0, \quad (13)$$

$$\lambda_t q(\theta_t)V_t = 0. \quad (14)$$

When the firm posts vacancies,  $V_t > 0$  and  $\lambda_t = 0$ . Equation (10) says that the marginal cost,  $\kappa$ , is equal to the marginal value of employment,  $\mu_t$ , conditional on the probability of a successful match,  $q(\theta_t)$ . When the nonnegativity constraint is binding,  $V_t = 0$  and  $\lambda_t > 0$ . It follows that  $\theta_t = V_t/U_t = 0$ , and from equation (3),  $q(\theta_t) = (1 + \theta_t^\epsilon)^{-1/\epsilon} = 1$ . In this case, the marginal value of employment  $\mu_t = \kappa - \lambda_t$ . All in all,  $\kappa/q(\theta_t) - \lambda_t$  can be interpreted as the marginal cost of vacancy posting, taking into account the matching probability and the nonnegativity constraint.

The intertemporal job creation condition (12) is intuitive. The marginal cost of vacancy posting at period  $t$  should be equal to the marginal benefit of vacancy posting at period  $t + 1$ , discounted to period  $t$  with the stochastic discount factor,  $M_{t+1}$ . The marginal benefit includes the marginal product of labor,  $X_{t+1}$ , net of the wage rate,  $W_{t+1}$ , and the marginal value of employment,  $\mu_{t+1}$ , which in turn equals the marginal cost of vacancy posting at  $t + 1$ , net of separation.

Define the stock return as  $R_{t+1} = S_{t+1}/(S_t - D_t)$  ( $S_t$  is the cum-dividend equity value). The



constant returns to scale assumption allows us to derive (see Appendix A for details):

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1-s)(\kappa/q(\theta_{t+1}) - \lambda_{t+1})}{\kappa/q(\theta_t) - \lambda_t}. \quad (15)$$

As such, the stock return is the ratio of the marginal benefit of vacancy posting at period  $t + 1$  over the marginal cost of vacancy posting at period  $t$ .

### 2.1.3 The Representative Family

The representative household maximizes utility, denoted  $J_t$ , over consumption using the Epstein and Zin preferences. The household can buy risky shares issued by the representative firm as well as a risk-free bond. Let  $C_t$  denote consumption and  $\chi_t$  denote the fraction of the household's wealth invested in the risky shares. The recursive utility function is given by:

$$J_t = \max_{\{C_t, \chi_t\}} \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left( E_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \quad (16)$$

in which  $\beta$  is time discount factor,  $\psi$  is the elasticity of intertemporal substitution, and  $\gamma$  is relative risk aversion. The Epstein-Zin preferences separate the elasticity of intertemporal substitution from the risk aversion. Intuitively,  $\psi$  measures the household's willingness to postpone consumption over time, and  $\gamma$  measures its aversion to atemporal risk across states. Separating the two parameters helps the model to produce a high equity premium and a low interest rate volatility simultaneously.

As noted, tradeable assets consist of risky shares and a risk-free bond. Let  $R_{t+1}^f$  denote the risk-free interest rate (known at the beginning of period  $t$ ) and  $R_{t+1}^\Pi$  the return on wealth, i.e.,  $R_{t+1}^\Pi = \chi_t R_{t+1} + (1 - \chi_t) R_{t+1}^f$ . Let  $\Pi_t$  denote the household's financial wealth,  $b$  the value of unemployment benefits,  $T_t$  the taxes raised by the government to pay for the unemployment benefits in lump-sum rebates. We can write the representative household's budget constraint as:

$$\frac{\Pi_{t+1}}{R_{t+1}^\Pi} = \Pi_t - C_t + W_t N_t + U_t b - T_t. \quad (17)$$

Note that the household's dividend income,  $D_t$ , is already included in the financial wealth,  $\Pi_t$ . (In

equilibrium,  $\Pi_t = S_t$ , the cum-dividend market value of equity, as shown in Section 2.1.5 later.) Finally, the government balances its budget, meaning that  $T_t = U_t b$ .

The household's first order condition with respect to the fraction of wealth invested in the risky asset,  $\chi_t$ , implies the fundamental equation of asset pricing:

$$1 = E_t[M_{t+1}R_{t+1}]. \quad (18)$$

In particular, the stochastic discount factor,  $M_{t+1}$ , is given by:

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}. \quad (19)$$

Finally, the risk-free rate is given by  $R_{t+1}^f = 1/E_t[M_{t+1}]$ .

#### 2.1.4 Wage Determination

The wage rate is determined endogenously as the outcome of a generalized Nash bargaining process between workers and the firm. Let  $0 < \eta < 1$  be the workers' relative bargaining weight. The wage equation is given by (see Appendix B for detailed derivations):

$$W_t = \eta[X_t + \kappa\theta_t] + (1 - \eta)b. \quad (20)$$

The wage rate is increasing in labor productivity,  $X_t$ , and in labor market tightness,  $\theta_t$ . Intuitively, the more productive the workers are, and the fewer workers there are relative to the number of vacancies, the higher the wage rates will be for the workers. Further, the value of unemployment activities,  $b$ , and the workers' bargaining weight,  $\eta$ , affect the elasticity of the wage with respect to productivity. The lower  $\eta$  is, and the higher  $b$  is, the more the wage will be tied with the constant unemployment value, inducing a higher degree of wage rigidity.

### 2.1.5 Competitive Equilibrium

In equilibrium, the financial markets clear. The risk-free asset is in zero net supply, and the household holds all the shares of the representative firm,  $\chi_t = 1$ . As such, the return on wealth equals the return on the firm,  $R_{t+1}^\Pi = R_{t+1}$ , and the household's wealth equals the cum-dividend equity value of the firm,  $\Pi_t = S_t$ . The goods market clearing condition implies the aggregate resource constraint:

$$C_t + \kappa V_t = X_t N_t. \quad (21)$$

The competitive equilibrium in the search economy consists of vacancy,  $V^* \geq 0$ ; multiplier,  $\lambda^* \geq 0$ ; consumption,  $C^*$ ; and indirect utility,  $J^*$ ; such that (i)  $V^*$  and  $\lambda^*$  satisfy the intertemporal job creation condition (12) and the Kuhn-Tucker condition (14), while taking the stochastic discount factor (19) and the wage equation (20) as given; (ii)  $C^*$  and  $J^*$  satisfy the intertemporal consumption-portfolio choice condition (18), in which the stock return is given by equation (15); and (iii) the goods market clears as in equation (21).

## 2.2 Calibration

We calibrate the model in monthly frequency. Table 1 lists the parameter values in the benchmark calibration. Following Gertler and Trigari (2009), we set the time discount factor,  $\beta$ , to be  $0.99^{1/3}$ , and the persistence of the aggregate productivity,  $\rho$ , to be  $0.95^{1/3} = 0.983$ . We choose the conditional volatility of the log aggregate productivity,  $\sigma$ , to be 0.0077 to target the standard deviation of output growth in the data. Following Bansal and Yaron (2004), we set the relative risk aversion,  $\gamma$ , to be 10, and the elasticity of intertemporal substitution,  $\psi$ , to be 1.5.

For the labor market parameters, we begin with the matching function,  $G(U_t, V_t)$ . Den Haan, Ramey, and Watson (2000) estimate the average monthly job filling rate to be  $\bar{q} = 0.71$  and the average monthly job finding rate to be  $\bar{f} = 0.45$  in the United States (see also Shimer (2005)). The constant returns to scale property of the matching function implies that the long-run average labor market tightness is around  $\bar{\theta} = \bar{f}/\bar{q} = 0.634$ . This estimate helps pin down the elasticity parameter

**Table 1 : Parameter Values in the Benchmark Calibration**

All the model parameters are calibrated at the monthly frequency.

Notation	Parameter	Value
$\beta$	Time discount factor	$0.99^{1/3}$
$\gamma$	Relative risk aversion	10
$\psi$	The elasticity of intertemporal substitution	1.5
$\rho$	Aggregate productivity persistence	0.983
$\sigma$	Conditional volatility of productivity shocks	0.0077
$\eta$	Workers' bargaining weight	0.10
$b$	The value of unemployment activities	0.85
$s$	Job separation rate	0.05
$\iota$	Elasticity of the matching function	1.290
$\kappa$	Cost of vacancy	0.975

in the matching function,  $\iota$ . Specifically, evaluating equation (3) at the long run average yields  $0.71 = (1 + 0.634^\iota)^{-1/\iota}$ , which in turn implies  $\iota = 1.29$ .

The average rate of unemployment for the United States over the 1920–2009 period is roughly 7%. However, important flows in and out of nonparticipation in the labor force as well as discouraged workers not accounted for in the pool of individuals seeking employment suggest that the unemployment rate should be calibrated higher. As such, we adopt the target average unemployment rate of  $\bar{U} = 10\%$ , which lies within the range between 7% in Gertler and Trigari (2009) and 12% in Krause and Lubik (2007). This target pins down the monthly job separation rate,  $s$ . In particular, the steady state labor market flows condition,  $s(1 - \bar{U}) = \bar{f}\bar{U}$ , sets  $s = 0.05$ . This value of  $s$ , which is also used by Andolfatto (1996), is close to the estimate of 0.053 from Clark (1990), and is within the range of estimates from Davis, Faberman, and Haltiwanger (2006).

The calibration of the value of unemployment activities,  $b$ , is controversial. On the one hand, Shimer (2005) pins down  $b = 0.4$  by assuming that the only benefit one gets while unemployed is government unemployment insurance benefits. Several subsequent studies such as Hall (2005) and Gertler and Trigari (2009) adopt such a conservative value for  $b$ , while exploring alternative wage

specifications that allow more stickiness in adjusting to labor productivity than the equilibrium wage emerged from the generalized Nash bargaining process. On the other hand, Hagedorn and Manovskii (2008) argue that in a perfect competitive labor market,  $b$  should equal the value of employment. In particular, the value of unemployment activities,  $b$ , measures not only unemployment insurance, but also the total value of leisure, home production, and self-employment. The long-run marginal product of labor in the model is unity, to which  $b$  should be close. Hagedorn and Manovskii choose a high value of 0.955 for  $b$ , which implies a low workers' bargaining power,  $\eta = 0.052$ , to match with the elasticity of the wage rate with respect to productivity around 0.45 in the data.

Our calibration of  $b$  is in the same spirit, but not as extreme as Hagedorn and Manovskii's (2008). In particular, we choose a value of 0.85 for  $b$ . Albeit high, this value is lower than 0.955 in Hagedorn and Manovskii. For the workers' bargaining weight,  $\eta$ , we set it to be 0.10. Because our calibration of  $b$  and  $\eta$  is not as extreme as Hagedorn and Manovskii's, the elasticity of the wage rate to productivity is 0.71 in the model, which is higher than that in the model.

With all the other parameters calibrated, we pin down the vacancy cost parameter,  $\kappa$ , by evaluating the intertemporal job creation condition (12) at the steady state. After we substitute the wage equation (20), the job creation condition at the steady state becomes:  $(1/\beta)(\kappa/\bar{q}) = \bar{X} - \eta(\bar{X} + \kappa\bar{\theta}) - (1 - \eta)b + (1 - s)(\kappa/\bar{q})$ , in which  $\bar{X} = 1$ . Solving for  $\kappa$  yields  $\kappa = (\bar{X} - b)(1 - \eta)\bar{q}/[1/\beta - (1 - s) + \eta\bar{\theta}\bar{q}]$ . This procedure produces a value of 0.975 for  $\kappa$ .

### 2.3 Nonlinear Solution Algorithm

Solving the model is challenging for several reasons. First, the search economy is not Pareto optimal, and the competitive equilibrium does not correspond to the social planner's solution. Intuitively, the firm in the decentralized economy does not take into account the congestion effect of posting a new vacancy on the labor market, whereas the social planner would do. As such, we need to solve for the competitive equilibrium directly. Second, because of the occasionally binding constraint (9), standard perturbation methods cannot be used. As such, we solve for the competitive equilibrium

using a nonlinear projection algorithm, while applying the Christiano and Fisher's (2000) idea of parameterized expectations to handle the constraint. Third, because of the nonlinearity inherent in the model and our focus on asset pricing moments that are sensitivity to nonlinearity, we need to solve the model on a large number of grid points to obtain an accurate proxy to the model's solution.

The state space of the model is  $(N_t, x_t)$ . The goal is to solve for the optimal vacancy function:  $V_t = V(N_t, x_t)$ , the multiplier function:  $\lambda_t = \lambda(N_t, x_t)$ , and an indirect utility function:  $J_t = J(N_t, x_t)$  from the following two functional equations:

$$J(N_t, x_t) = \left[ (1 - \beta)C(N_t, x_t)^{1 - \frac{1}{\psi}} + \beta \left( E_t [J(N_{t+1}, x_{t+1})^{1 - \gamma}] \right)^{\frac{1 - 1/\psi}{1 - \gamma}} \right]^{\frac{1}{1 - 1/\psi}} \quad (22)$$

$$\frac{\kappa}{q(\theta_t)} - \lambda(N_t, x_t) = E_t \left[ M_{t+1} \left[ X_{t+1} - W_{t+1} + (1 - s) \left( \frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1}) \right) \right] \right]. \quad (23)$$

$V(N_t, x_t)$  and  $\lambda(N_t, x_t)$  must also satisfy the Kuhn-Tucker condition:  $V_t \geq 0, \lambda_t \geq 0$ , and  $\lambda_t V_t = 0$ .

The traditional projection method would parameterize  $V(N_t, x_t)$  and  $\lambda(N_t, x_t)$ , such that the parameterized functions solve the two functional equations, while obeying the Kuhn-Tucker condition. As pointed out by Christiano and Fisher (2000), with the occasionally binding constraint, this approach is tricky and cumbersome. Instead, we follow Christiano and Fisher in parameterizing the conditional expectation function in the job creation condition (23). Specifically, we parameterize:

$$\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t) = E_t \left[ M_{t+1} \left[ X_{t+1} - W_{t+1} + (1 - s) \left( \frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1}) \right) \right] \right]. \quad (24)$$

We then exploit a convenient mapping from the conditional expectation function to policy and multiplier functions, thereby eliminating the need to separately parameterize the multiplier function. Specifically, after obtaining the parameterized  $\mathcal{E}_t$ , we first calculate:

$$\bar{q}(\theta_t) = \frac{\kappa}{\mathcal{E}_t}. \quad (25)$$

If  $\bar{q}(\theta_t) < 1$ , the nonnegativity constraint is not binding, we set  $\lambda_t = 0$  and  $q(\theta_t) = \bar{q}(\theta_t)$ . We solve  $\theta_t = q^{-1}(\kappa/\mathcal{E}_t)$ , in which  $q^{-1}(\cdot)$  is the inverse function of  $q(\cdot)$  given by equation (3), and

$V_t = \theta_t(1 - N_t)$ . If  $\bar{q}(\theta_t) \geq 1$ , the nonnegativity constraint is binding, we set  $V_t = 0$ ,  $\theta_t = 0$ ,  $q(\theta_t) = 1$ , and  $\lambda_t = \kappa - \mathcal{E}_t$ . This approach is convenient in practice because it enforces the Kuhn-Tucker condition automatically so as to eliminate the need of parameterizing the multiplier function.

We approximate the  $x_t$  process in equation (5) based on the discrete state space method of Rouwenhorst (1995) with 15 grid points.<sup>1</sup> The  $x$ -grid is large enough to cover  $x$ -values that are within four unconditional standard deviations from its unconditional mean. We set the minimum value of  $N_t$  to be 0.03 and the maximum value to be 0.99. This range is large enough so that in simulations  $N_t$  never hits one of the boundaries. We use cubic splines with 40 basis functions on the  $N$  space to approximate  $\mathcal{E}(N_t, x_t)$  on each grid point of  $x$ .<sup>2</sup> To obtain an initial guess for the projection algorithm, we use the social planner's solution via value function iteration. We apply the homotopy idea extensively to visit the parameter space in which the model exhibits strong nonlinearity.

Figure 1 reports the error in the  $J$  functional equation (22), defined as  $J(N_t, x_t)^{1-\frac{1}{\psi}} - (1 - \beta)C(N_t, x_t)^{1-\frac{1}{\psi}} - \beta \left( E_t \left[ J(N_{t+1}, x_{t+1})^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}}$ , and the error in the  $\mathcal{E}$  functional equation (24), defined as  $\mathcal{E}(N_t, x_t) - E_t \left[ M_{t+1} \left[ X_{t+1} - W_{t+1} + (1 - s) \left( \frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1}) \right) \right] \right]$ . These errors, in the magnitude no higher than  $10^{-14}$ , are extremely small. As such, our nonlinear algorithm does an accurate job in characterizing the competitive equilibrium in the search economy.

## 2.4 The Model's Properties

Before studying the model's quantitative results, we first examine qualitative properties of the solution on the grid of employment and productivity (Appendix D contains more details).

Panel A of Figure 2 shows that labor market tightness,  $\theta_t$ , is increasing in employment and in aggregate productivity. The labor market is tighter from the perspective of the firm when there are fewer unemployed workers searching for jobs (employment is high), and when the demand for workers is high (labor productivity is high). Panel B confirms that the labor market is more congested

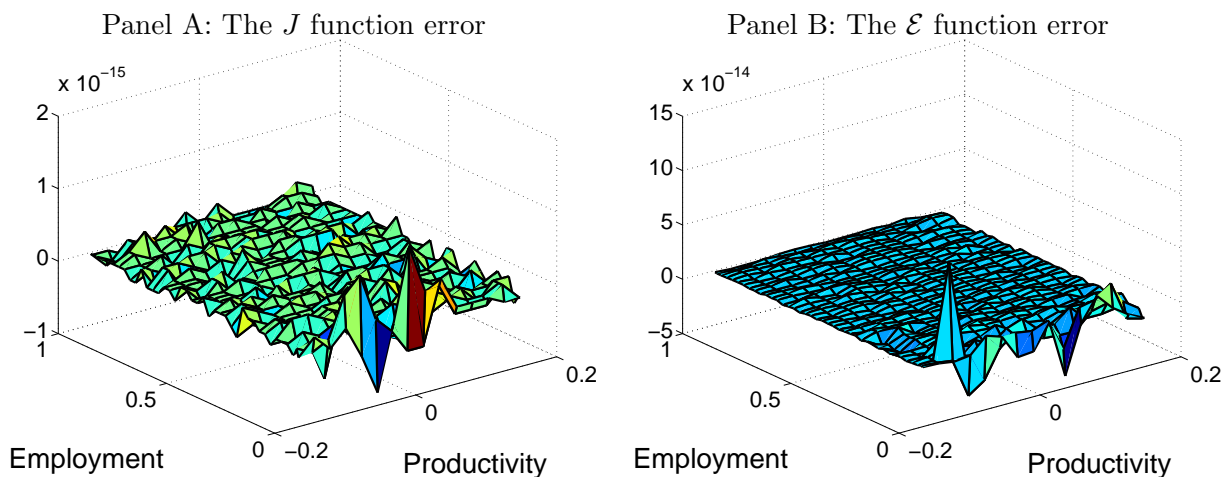
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<sup>1</sup>Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more reliable and accurate than other methods in approximating highly persistent first-order autoregressive processes.

<sup>2</sup>We use extensively the approximation tool kit in the CompEcon Toolbox of Miranda and Fackler (2002).

**Figure 1 : Errors in the  $J$  and  $\mathcal{E}$  Functional Equations**

The  $J$  function error is  $J(N_t, x_t)^{1-\frac{1}{\psi}} - (1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} - \beta \left( E_t [J(N_{t+1}, x_{t+1})^{1-\gamma}] \right)^{\frac{1-1/\psi}{1-\gamma}}$ , and the  $\mathcal{E}$  function error is  $\mathcal{E}(N_t, x_t) - E_t \left[ M_{t+1} \left[ X_{t+1} - W_{t+1} + (1-s) \left( \frac{\kappa}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1}) \right) \right] \right]$ . We plot the errors on the two-dimensional grid of  $N_t$  (employment) and  $x_t$  (productivity).



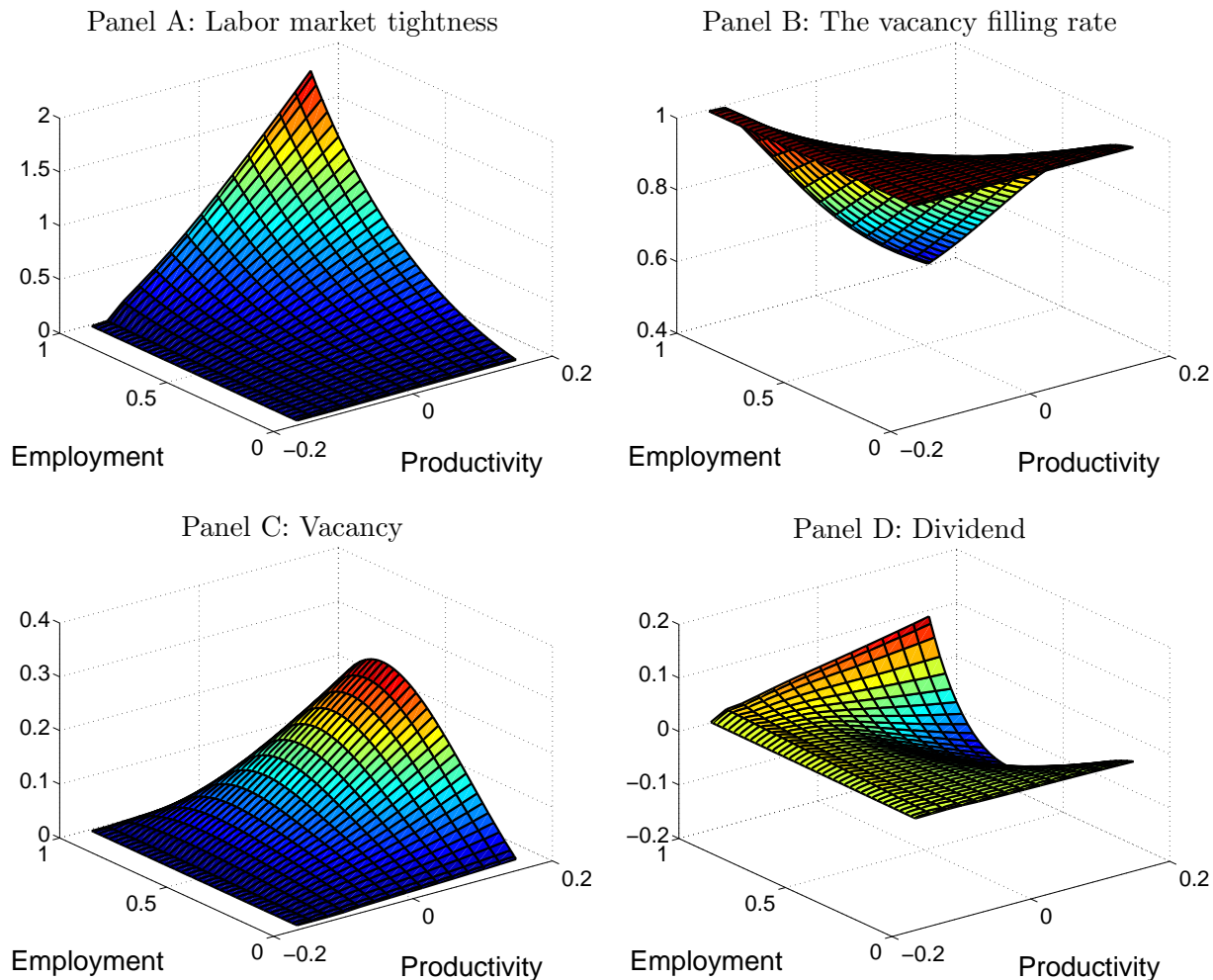
in the states with high employment and high productivity. The vacancy fill rate,  $q(\theta_t)$ , is the lowest in these states, but hits the maximum of unity in the low-employment-low-productivity states.

From Panel C of Figure 2, the optimal vacancy is procyclical (increasing with aggregate productivity), but is hump-shaped in employment. Intuitively, operating profits,  $X_t N_t - W_t N_t$ , increase with labor productivity, meaning that the firm posts more vacancies in good times. Conditional on productivity, low employment means low operating profits. As such, the firm posts fewer vacancies to avoid vacancy costs that are high relative to profits. At the other extreme, although high employment means high operating profits, it also implies more congestion and low vacancy filling rates,  $q(\theta_t)$ , which the firm takes as given. The congestion deters the firm from posting more vacancies.

Panel D shows that the dynamics of dividend are more complex. First, the dividend dynamics are closely related to the vacancy dynamics. When productivity is high, dividend shows a U-shape in employment. Dividend is positive and procyclical when employment is around its long run average (around 0.90). But dividend can be negative and countercyclical when employment is in the



**Figure 2 : Labor Market Tightness, the Vacancy Filling Rate, Vacancy, and Dividend**

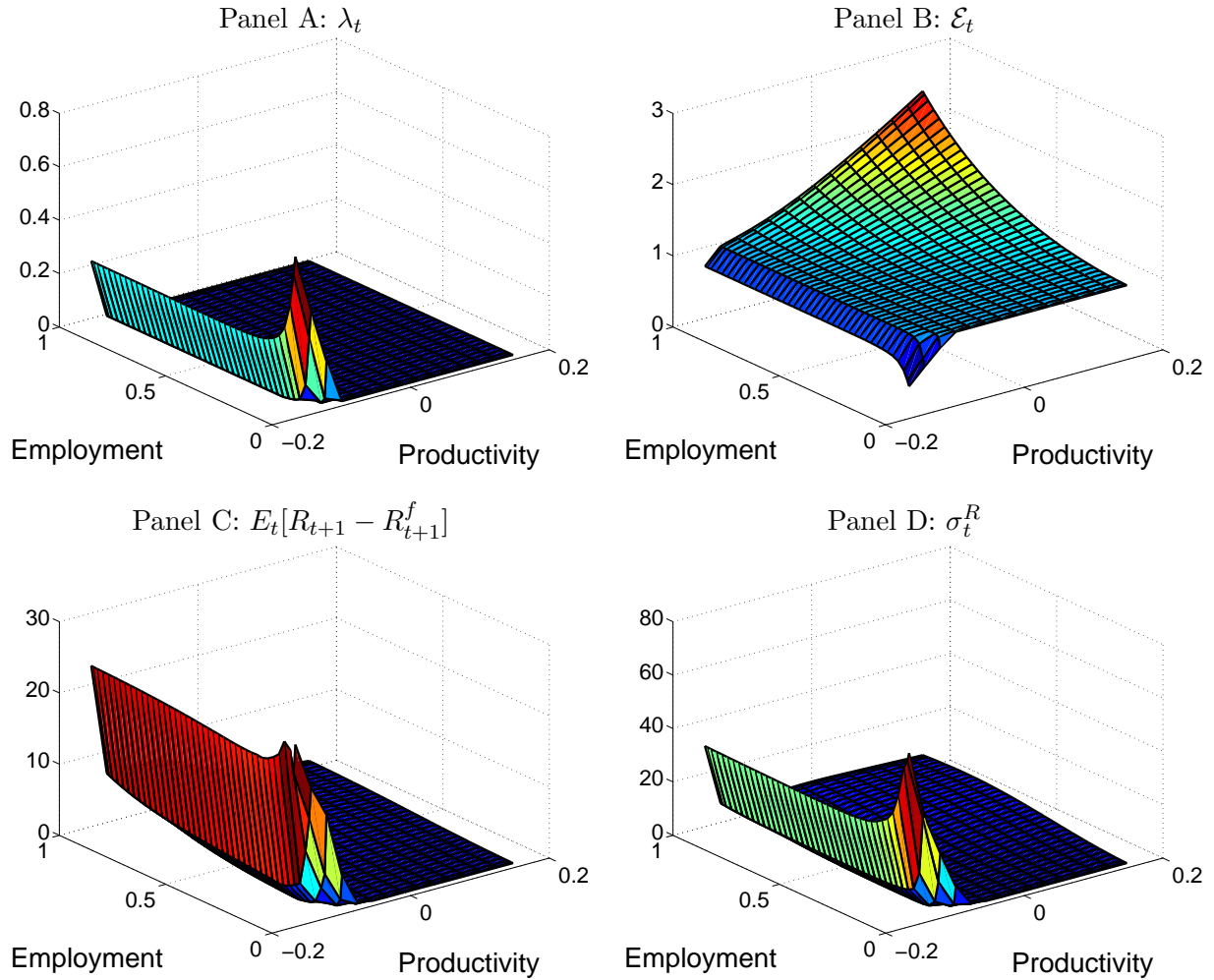


middle range on the  $N$  grid. Negative dividends can be seen as positive equity issues in the model. The countercyclical of dividend is the mirror image of the procyclicality of vacancy because the firm posts more vacancies in the middle range of the  $N_t$  grid.

Panel A of Figure 3 plots the multiplier on the nonnegativity constraint on vacancy,  $\lambda_t$ . We see that  $\lambda_t$  is countercyclical.  $\lambda_t$  equals zero for most values of productivity, but increases rapidly as productivity approaches its lowest level. The multiplier is also convex in employment.  $\lambda_t$  is flat across most values of employment, but rises with an increasing speed as it approaches its lowest level.

To obtain some intuition, Panel B plots the conditional expectation,  $\mathcal{E}_t$ , defined in equation (24). As noted, when the nonnegativity constraint is binding,  $V_t = 0, \theta_t = 0$ , and  $q(\theta_t) = 1$ . As such,

**Figure 3 : The Multiplier on the Nonnegativity Constraint of Vacancy,  $\lambda_t$ , the Conditional Expectation in the Intertemporal Job Creation Condition,  $\mathcal{E}_t$ , the Equity Risk Premium in Annual Percent,  $E_t[R_{t+1} - R_{t+1}^f]$ , and the Conditional Market Volatility in Annual Percent,  $\sigma_t^R$**



$\lambda_t = \kappa - \mathcal{E}_t$ . From Panel B,  $\mathcal{E}_t$  is increasing in both employment and productivity. More important,  $\mathcal{E}_t$  shows strong nonlinearity as it drops rapidly as the economy approaches the corner with the lowest levels of employment and productivity simultaneously. This nonlinearity is a natural result of the stochastic discount factor,  $M_{t+1}$ . As consumption approaches zero, marginal utility blows up, causing  $\mathcal{E}_t$  to drop and  $\lambda_t$  to rise precipitously.

Panels C and D report two key financial moments, the equity risk premium and the conditional market volatility. Both moments exhibit similar dynamics as the multiplier on the occasionally binding constraint,  $\lambda_t$ . In particular, both moments are strongly countercyclical, and are largely convex in employment as it approaches its lowest level.

### 3 Quantitative Results

We present basic business cycle and asset pricing moments in Section 3.1 and labor market moments in Section 3.2. In Section 3.3, we examine the linkage between the labor market and the financial market by using labor market tightness to forecast stock market excess returns. In Section 3.4, we quantify the model's endogenous rare disaster risks that are important for equilibrium asset prices.

#### 3.1 Basic Business Cycle and Financial Moments

Panel A of Table 2 reports the standard deviation and autocorrelations of (log) consumption growth and (log) output growth, as well as unconditional financial moments in the data. Consumption is annual real personal consumption expenditures, and output is annual real gross domestic product from 1929 to 2010 from Bureau of Economic Analysis at U.S. Department of Commerce. The annual consumption growth in the data has a volatility of 3.04%, and a first-order autocorrelation of 0.38. The autocorrelation drops to 0.08 at the two-year horizon, and turns negative,  $-0.21$ , at the three-year horizon. The annual output growth has a volatility of 4.93% and a high first-order autocorrelation of 0.54. The autocorrelation drops to 0.18 at the two-year horizon, and turns negative afterward:  $-0.18$  at the three-year horizon and  $-0.23$  at the five-year horizon.

**Table 2 : Basic Business Cycle and Financial Moments**

In Panel A, consumption is annual real personal consumption expenditures (series PCECCA), and output is annual real gross domestic product (series GDPCA) from 1929 to 2010 (82 annual observations) from Bureau of Economic Analysis at U.S. Department of Commerce.  $\sigma^Y$  is the volatility of log output growth, and  $\sigma^C$  is the volatility of log consumption growth. Both volatilities are in percent.  $\rho^C(\tau)$  and  $\rho^Y(\tau)$ , for  $\tau = 1, 2, 3$ , and 5, are the  $\tau$ -th order autocorrelations of log consumption growth and log output growth, respectively. We obtain monthly series from January 1926 to December 2010 (1,020 monthly observations) for the value-weighted market index returns including dividends, one-month Treasury bill rates, and the rates of change in Consumer Price Index (inflation rates) from CRSP.  $E[R - R^f]$  is the average (in annualized percent) of the value-weighted market returns in excess of the one-month Treasury bill rates, adjusted for the long-term market leverage rate of 0.32 reported by Frank and Goyal (2008). (The leverage-adjusted average  $E[R - R^f]$  is the unadjusted average times 0.68.)  $E[R^f]$  and  $\sigma^{R^f}$  are the mean and volatility, both of which are in annualized percent, of real interest rates, defined as the one-month Treasury bill rates in excess of the inflation rates.  $\sigma^R$  is the volatility (in annualized percent) of the leverage-weighted average of the value-weighted market returns in excess of the inflation rates and the real interest rates. In Panel B, we simulate 1,000 artificial samples, each of which has 1,020 monthly observations, from the model in Section 2. In each artificial sample, we calculate the mean market excess return,  $E[R - R^f]$ , the volatility of the market return,  $\sigma^R$ , as well as the mean,  $E[R^f]$ , and volatility,  $\sigma^{R^f}$ , of the real interest rate. All these moments are in annualized percent. We also time-aggregate the first 984 monthly observations of consumption and output into 82 annual observations in each sample, and calculate the annual volatilities and autocorrelations of log consumption growth and log output growth. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is larger than its data moment.

	Panel A: Data	Panel B: Model			
		Mean	5%	95%	p-value
$\sigma^Y$	4.933	4.945	3.024	8.639	0.342
$\rho^Y(1)$	0.543	0.236	0.018	0.521	0.038
$\rho^Y(2)$	0.178	-0.127	-0.324	0.112	0.025
$\rho^Y(3)$	-0.179	-0.131	-0.337	0.113	0.636
$\rho^Y(5)$	-0.227	-0.082	-0.295	0.135	0.866
$\sigma^C$	3.036	4.643	2.691	8.473	0.846
$\rho^C(1)$	0.383	0.243	0.013	0.530	0.168
$\rho^C(2)$	0.081	-0.131	-0.328	0.110	0.068
$\rho^C(3)$	-0.206	-0.135	-0.346	0.111	0.697
$\rho^C(5)$	0.062	-0.082	-0.301	0.140	0.145
$E[R - R^f]$	5.066	3.669	2.951	4.324	0.000
$E[R^f]$	0.588	3.751	3.235	4.143	1.000
$\sigma^R$	12.942	7.829	7.147	8.572	0.000
$\sigma^{R^f}$	1.872	1.439	0.928	2.267	0.125

We obtain monthly series of the value-weighted market returns including all NYSE, Amex, and Nasdaq stocks, one-month Treasury bill rates, and inflation rates (the rates of change in Consumer Price Index) from Center for Research in Security Prices (CRSP). The sample is from January 1926 to December 2010 (1,020 months). The mean of real interest rates (one-month Treasury bill rates minus inflation rates) is 0.59% per annum, and the annualized volatility is 1.87%. The equity premium (the average of the value-weighted market returns in excess of one-month Treasury bill rates) in the 1926–2010 sample is 7.45% per annum. Because we do not model financial leverage, we adjust the equity premium in the data for leverage before matching with the equity premium implied from the model. Frank and Goyal (2008) report that the aggregate market leverage ratio of U.S. corporations is fairly stable around 0.32. As such, we calculate the leverage-adjusted equity premium as  $(1 - 0.32) \times 7.45\% = 5.07\%$  per annum. The annualized volatility of the market returns in excess of inflation rates is 18.95%. Adjusting for leverage (taking the leverage-weighted average of real market returns and real interest rates) yields an annualized volatility of 12.94%.

Panel B of Table 2 reports the model moments. From the initial condition of zero for aggregate productivity,  $x_t$ , and 0.90 for employment,  $N_t$ , we first simulate the economy for 6,000 monthly periods to reach its stationary distribution. We then repeatedly simulate 1,000 artificial samples, each with 1,020 months. On each artificial sample, we calculate the annualized monthly averages of the equity premium and the real interest rate, as well as the annualized monthly volatilities of the market returns and the real interest rate. We also take the first 984 monthly observations of consumption and output, and time-aggregate them into 82 annual observations. (We add up 12 monthly observations within a given year, and treat the sum as the year’s annual observation.) For each data moment, we report the average as well as the 5 and 95 percentiles across the 1,000 simulations. The p-values are the frequencies with which a given model moment is larger than its data counterpart.

Because we calibrate the volatility of the log productivity shocks,  $\sigma$ , to target the (log) output growth volatility,  $\sigma^Y$ , the model implied output growth volatility is 4.95% per annum, which is close to 4.93% in the data. The model reproduces a positive first-order autocorrelation of 0.24 for the

output growth, but is lower than 0.54 in the data. Also, the model implies a negative second-order autocorrelation of  $-0.13$ , but this autocorrelation is 0.18 in the data. Both correlations in the data are outside the 90% confidence interval of the model's bootstrapped distribution. At the longer horizons, the autocorrelations are both negative in the model, consistent with the data.

The model predicts a consumption growth volatility of 4.64% per annum, which is somewhat higher than 3.04% in the data. However, this data moment lies within the 90% confidence interval of the model's bootstrapped distribution with a bootstrapped p-value of 0.85. The model also implies a positive first-order autocorrelation of 0.24 for consumption growth and negative autocorrelations at longer horizons. All the autocorrelations in the data are within 90% confidence interval of the model.

The model's performance in matching financial moments seems fair. The equity premium is 3.67% per annum, and the market volatility is 7.83%. The real interest rate is on average 3.75%, and its volatility is 1.44%. Although all the data moments except for the interest rate volatility are outside the model's 90% confidence interval, we view the fit as solid progress in the context of aggregate asset pricing with production (see the references discussed in Section 1).

### 3.2 Labor Market Moments

To evaluate the model's fit for labor market moments, we first document these moments per Hagedorn and Manovskii (2008, Table 3) using an updated sample. We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the Bureau of Labor Statistics (BLS) at the U.S. Department of Labor, and seasonally adjusted help wanted advertising index from the Conference Board. The sample is from January 1951 to June 2006.<sup>3</sup> We take quarterly averages of the monthly series to obtain quarterly observations. The average labor productivity is seasonally adjusted real average output per person in the nonfarm business sector from BLS.

Hagedorn and Manovskii (2008) report all variables in logs as deviations from the Hodrick-

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<sup>3</sup>The sample ends in June 2006 because the Conference Board switches from help wanted advertising index to help wanted online index around that time. The two indexes are not directly comparable. As such, we follow the standard practice in the labor literature to use the longer time series before the switch.

Prescott (1997, HP) trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean (with the same smoothing parameter).<sup>4</sup> We do not use log deviations because vacancies can be zero in the model’s simulations when the nonnegativity constraint is binding. In the data, the two detrending methods yield quantitatively similar results, which are in turn close to Hagedorn and Manovskii’s. In particular, from Panel A of Table 3, the standard deviation of the  $V/U$  ratio is 0.26. The  $V/U$  ratio is also procyclical with a positive correlation of 0.30 with labor productivity in the data. Finally, vacancy and unemployment have a negative correlation of  $-0.91$ .

To evaluate the model’s fit with the labor market moments, we simulate 1,000 artificial samples, each with 666 months. We take the quarterly averages of the monthly unemployment,  $U$ , vacancy,  $V$ , and labor productivity,  $X$ , to obtain 222 quarterly observations for each series. We then apply the exactly same procedures as in Panel A of Table 3 on the artificial data, and report the cross-simulation averages (as well as standard deviations) for the model moments.

Panel B reports the model’s quantitative performance. The model implies a standard deviation of 0.16 for the  $V/U$  ratio, which is lower than 0.26 in the data. The bootstrapped standard deviation of this model moment is 0.03. As such, the data moment is more than three standard deviations away from the model moment. This result is consistent with Shimer’s (2005) critique that the standard search model fails to explain the volatility of labor market tightness in the data.

The standard deviations of  $U$  and  $V$  in the model are 0.14 and 0.10, which are close to those in the data. The model also generates a Beveridge curve with a negative  $U-V$  correlation of  $-0.51$ . However, its magnitude is lower than  $-0.91$  in the data. The correlation between the  $V/U$  ratio and labor productivity is 0.99 in the model, which is higher than 0.30 in the data. Further, the model implies an average unemployment rate of 0.12, an job finding rate of 0.42, a vacancy filling rate of 0.73, and an average  $V/U$  ratio of 0.61. (Because of the model’s nonlinearity, the long-run

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<sup>4</sup>Specifically, for any variable  $Z$ , the HP-filtered cyclical component of proportional deviations from the mean is calculated as  $(Z - \bar{Z})/\bar{Z} - \text{HP}[(Z - \bar{Z})/\bar{Z}]$ , in which  $\bar{Z}$  is the mean of  $Z$ , and  $\text{HP}[(Z - \bar{Z})/\bar{Z}]$  is the HP trend of  $(Z - \bar{Z})/\bar{Z}$ .

**Table 3 : Labor Market Moments**

In Panel A, seasonally adjusted monthly unemployment ( $U$ , thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index,  $V$ , is from the Conference Board. The series are monthly from January 1951 to June 2006 (666 months). Both  $U$  and  $V$  are converted to quarterly averages of monthly series. The average labor productivity,  $X$ , is seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. For example, for the variable  $X$ , its cyclical component is calculated as  $(X - \bar{X})/\bar{X} - \text{HP}[(X - \bar{X})/\bar{X}]$ , in which  $\bar{X}$  is the mean of  $X$ , and  $\text{HP}[(X - \bar{X})/\bar{X}]$  is the HP trend of  $(X - \bar{X})/\bar{X}$ . In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. We take the quarterly averages of monthly  $U, V$ , and  $X$  to convert to 222 quarterly observations. The exactly same empirical procedures as in Panel A are then implemented on these quarterly series. We report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

Panel A: Data					
	$U$	$V$	$V/U$	$X$	
Standard deviation	0.119	0.134	0.255	0.012	
Quarterly autocorrelation	0.902	0.922	0.889	0.761	
Correlation matrix	1	-0.913	-0.801	-0.224	$U$
		1	0.865	0.388	$V$
			1	0.299	$V/U$
				1	$X$
Panel B: Model					
	$U$	$V$	$V/U$	$X$	
Standard deviation	0.143 (0.051)	0.095 (0.019)	0.158 (0.030)	0.016 (0.002)	
Quarterly autocorrelation	0.879 (0.040)	0.617 (0.066)	0.804 (0.036)	0.773 (0.040)	
Correlation matrix	1	-0.510 (0.081)	-0.704 (0.116)	-0.655 (0.128)	$U$
		1	0.857 (0.055)	0.915 (0.023)	$V$
			1	0.988 (0.018)	$V/U$
				1	$X$



average unemployment rate in simulations is somewhat higher than the calibration target of 10%, which is only based on steady state relations.) These moments are close to those in the data.

### 3.3 The Linkage between the Labor Market and the Financial Market

A large literature in financial economics shows that the equity risk premium is time-varying (and countercyclical) in the data (e.g., Lettau and Ludvigson (2001)). In the labor market, as vacancy is procyclical and unemployment is countercyclical, the  $V/U$  ratio (labor market tightness) is strongly procyclical (e.g., Shimer (2005)). As such, the  $V/U$  ratio should forecast stock market excess returns with a negative slope. Panel A of Table 4 documents such predictability in the data.

Specifically, we perform monthly long-horizon regressions of log excess returns on the CRSP value-weighted market returns,  $\sum_{h=1}^H R_{t+3+h} - R_{t+3+h}^f$ , in which  $H = 1, 3, 6, 12, 24,$  and  $36$  is the forecast horizon in months. When  $H > 1$ , we use overlapping monthly observations of  $H$ -period holding returns. The regressors are *two-month* lagged values of the  $V/U$  ratio.<sup>5</sup> The BLS takes less than one week to release monthly employment and unemployment data, and the Conference Board takes about one month to release monthly help wanted advertising index data.<sup>6</sup> We impose the two-month lag between the  $V/U$  ratio and market excess returns to guard against look-ahead bias in predictive regressions. To make the regression slopes comparable to those in the model, we also scale up the  $V/U$  series in the data by a factor of 50 to make its average close to that in the model. This scaling is necessary because the vacancy and unemployment series in the data have different units.

Panel A of Table 4 shows that the  $V/U$  ratio is a reliable negative forecaster of market excess returns at business cycle frequencies. For example, at the one-month horizon, the slope is  $-1.43$ , which is more than 2.5 standard errors from zero. (The standard errors are adjusted for heteroscedasticity and autocorrelations of 12 lags per Newey and West (1987)). The adjusted  $R^2$  is close to 1%. The slopes remain significant at the three-month and six-month horizons, but become

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<sup>5</sup>Per our timing convention, returns such as  $R_{t+1}$  are observed at the end of period  $t$ , but labor market tightness,  $V_t/U_t$ , is observed at the beginning of period  $t$ , even though  $V_t$  occurs during the course of period  $t$ .

<sup>6</sup>We verify this practice through a private correspondence with the Conference Board staff.

**Table 4 : Long-Horizon Regressions of Market Excess Returns on Labor Market Tightness**

Panel A reports long-horizon regressions of log excess returns on the value-weighted market index from CRSP,  $\sum_{h=1}^H R_{t+3+h} - R_{t+3+h}^f$ , in which  $H$  is the forecast horizon in months. The regressors are two-month lagged values of the  $V/U$  ratio. We report the ordinary least squares estimate of the slopes (Slope), the Newey-West corrected  $t$ -statistics ( $t_{NW}$ ), and the adjusted  $R^2$ s. The seasonally adjusted monthly unemployment ( $U$ , thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics, and the seasonally adjusted help wanted advertising index ( $V$ ) is from the Conference Board. The sample is from January 1951 to June 2006 (666 monthly observations). We multiply the  $V/U$  series by 50 so that its average is close to that in the model. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. On each artificial sample, we implement the exactly same empirical procedures as in Panel A, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Panel A: Data						
Slope	-1.425	-4.203	-7.298	-10.312	-9.015	-10.156
$t_{NW}$	-2.575	-2.552	-2.264	-1.704	-0.970	-0.861
Adjusted $R^2$	0.950	2.598	3.782	3.672	1.533	1.405
Panel B: Model						
Slope	-0.758	-2.231	-4.321	-8.108	-14.407	-19.463
	(0.456)	(1.329)	(2.553)	(4.767)	(8.282)	(10.866)
$t_{NW}$	-1.929	-2.021	-2.131	-2.377	-2.977	-3.503
	(0.850)	(0.903)	(0.977)	(1.161)	(1.555)	(1.889)
Adjusted $R^2$	0.555	1.628	3.130	5.859	10.419	14.142
	(0.433)	(1.245)	(2.361)	(4.375)	(7.575)	(10.013)

insignificant afterward. The adjusted  $R^2$  peaks at 3.78% at the six-month horizon, and declines to 3.67% at the one-year horizon and further to 1.41% at the three-year horizon.

Panel B of Table 4 reports the model's fit for the predictive regressions. Consistent with the data, the model predicts that the  $V/U$  ratio forecasts market excess returns with a negative slope. In particular, at the one-month horizon, the predictive slope is  $-0.76$  with a  $t$ -statistic of  $-1.93$ . At the six-month horizon, the slope is  $-4.32$  with a  $t$ -statistic of  $-2.13$ . The slopes in the model are smaller in magnitude than those in the data because the equity risk premium in the model is lower. However, the model implies stronger predictive power for the  $V/U$  ratio than that in the data. Both the  $t$ -statistics and the adjusted  $R^2$ s increase monotonically with the forecast horizon.

In contrast, both measures peak at the six-month horizon and decline afterward in the data.

Panel A of Figure 4 reports further evidence on time-varying risk premiums. We plot the cross-correlations (and their two standard-error bounds) between labor market tightness,  $V_t/U_t$ , and future market excess returns,  $R_{t+H} - R_{t+H}^f$ , for  $H = 1, 2, \dots, 36$  months in the data. No overlapping observations are used in calculating these cross-correlations. The panel shows that the correlations are negative and significant for forecast horizons up to six months. The evidence is consistent with the predictive regressions in Table 4.

Panel B reports the cross-correlations (and their two cross-simulation standard-deviation bounds) from the model's bootstrapped distribution. Consistent with the data, the model predicts significantly negative cross-correlations between the  $V/U$  ratio and future market excess returns for horizons up to about six months. However, although the cross-correlations are insignificant beyond the six-month horizon, the correlations decay more slowly over the forecast horizon than those in the data. This (counterfactual) pattern is also consistent with Panel B of Table 4, which shows that the predictive power of the  $V/U$  ratio in the model increases over the forecast horizon.

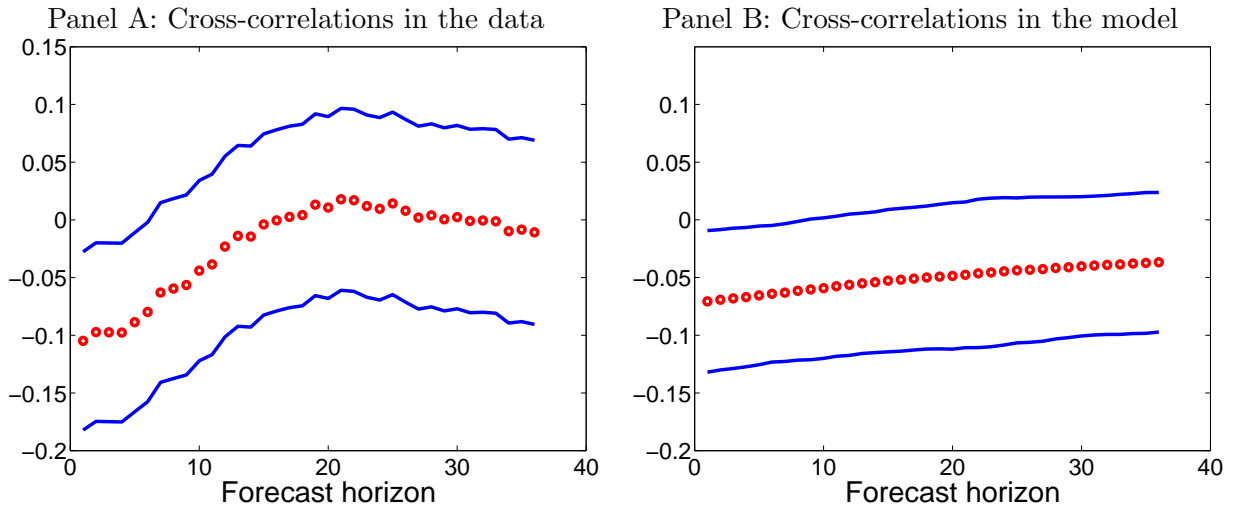
### 3.4 Endogenous Rare Disasters

The search economy gives rise endogenously to rare disaster risks à la Rietz (1988) and Barro (2006). To see this point, we simulate 1,006,000 monthly periods from the model, discard the first 6,000 periods, and treat the remaining one million months as the model's stationary distribution. Figure 5 reports the empirical cumulative distribution functions, denoted *ECDF*, for unemployment, output, consumption, dividend, the equity risk premium, and vacancy.

The economy exhibits infrequent but deep crashes. From Panel A of Figure 5, unemployment is positively skewed with a long right tail. As the population moments, the mean unemployment rate is 12.14%, the median is 10.29%, and the skewness is 5.09. The 2.5 percentile of unemployment is close to the median, 7.44%, whereas the 97.5 percentile is far away from the median, 29.10%. As a mirror image, the employment rate is negatively skewed with a long left tail. As a result,

**Figure 4 : Cross-Correlations between the  $V/U$  Ratio and Future Market Excess Returns**

We report the cross-correlations between labor market tightness,  $V_t/U_t$ , and future market excess returns,  $R_{t+H} - R_{t+H}^f$ , in which  $H = 1, 2, \dots, 36$  is the forecast horizon in months. We report the cross-correlations both in the data (Panel A) and in the model (Panel B). In Panel A,  $V_t$  is the seasonally adjusted help wanted advertising index from the Conference Board, and  $U_t$  is the seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the BLS. The sample is from January 1951 to June 2006. The stock market excess returns are the CRSP value-weighted market returns in excess of one-month Treasury bill rates. The two standard errors bounds for the cross-correlations are also plotted. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. On each artificial sample, we calculate the cross-correlations between  $V_t/U_t$  and  $R_{t+H} - R_{t+H}^f$ , and plot the cross-simulation averaged correlations as well as their two cross-simulation standard errors bounds.

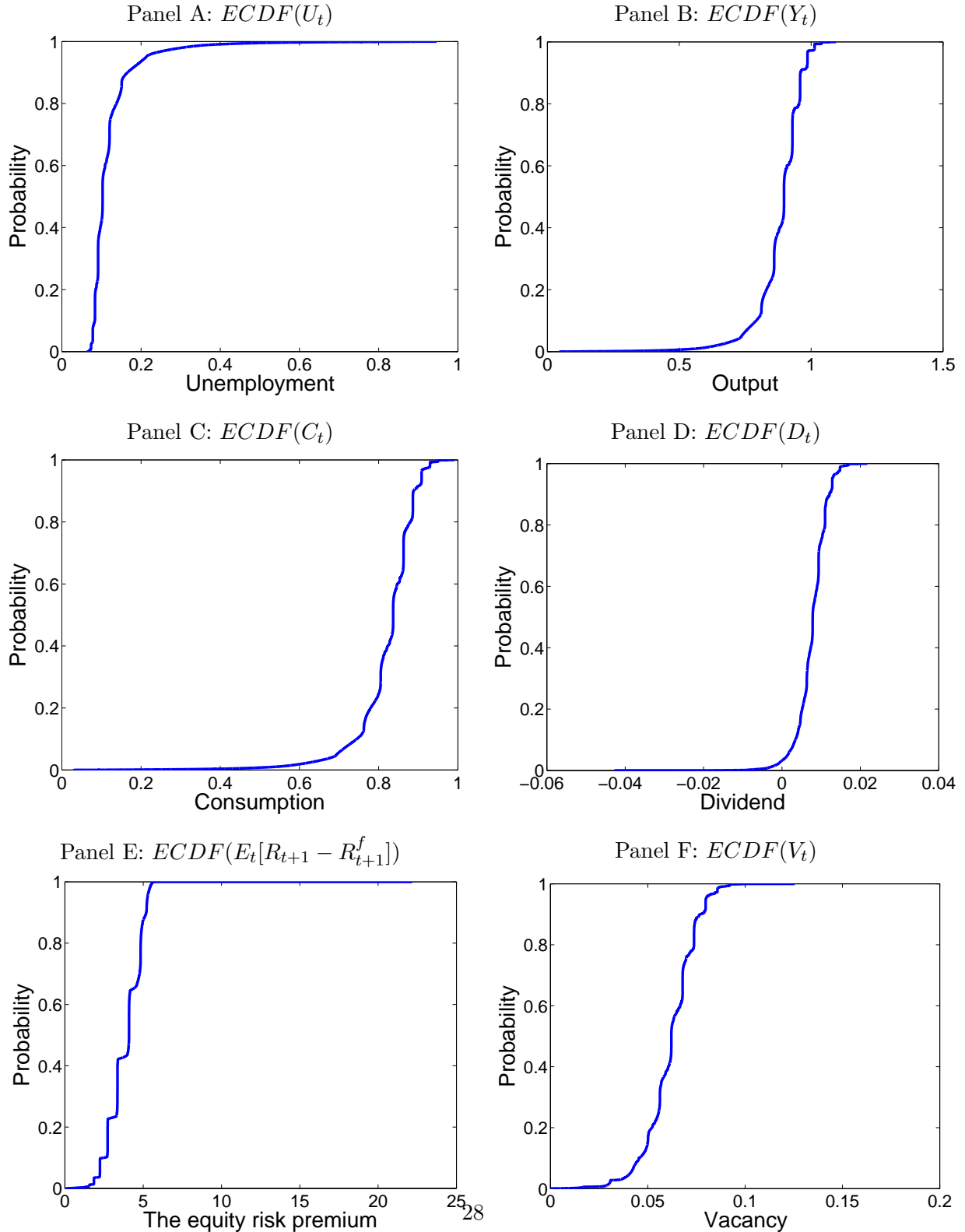


output, consumption, and dividend all show infrequent but severe disasters (Panels B–D). With small probabilities, the economy enters deep depressions in simulations.

The disasters in macroeconomic dynamics reflect in asset prices as rare upward spikes in the equity risk premium. From Panel E of Figure 5, the stationary distribution of the equity risk premium is positively skewed with a long right tail. The mean equity premium is 3.87% per annum, and its 2.5 and 97.5 percentiles are 1.86% and 5.40%, respectively. However, with small probabilities, the risk premium can reach very high levels. In particular, the 99.99 percentile is 15.51%. Finally, Panel F shows that the cumulative distribution function of vacancy is largely symmetrical. Labor

**Figure 5 : Empirical Stationary Distribution of the Model: Unemployment, Output, Consumption, Dividend, the Equity Risk Premium, and Vacancy**

We simulate 1,006,000 monthly periods from the model, discard the first 6,000 periods, and treat the remaining one million months as from the model's stationary distribution. We plot the empirical cumulative distribution functions,  $ECDF(\cdot)$ , for unemployment,  $U_t$ , output,  $Y_t$ , consumption,  $C_t$ , dividend,  $D_t$ , the equity risk premium,  $E_t[R_{t+1} - R_{t+1}^f]$ , and vacancy,  $V_t$ .



market tightness, the vacancy filling rate, and wage all display similar distributions (untabulated).

Barro and Ursúa (2008) apply a peak-to-trough method on samples from 1870 to 2006 to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. Suppose there are two states, normalcy and disaster. The disaster probability measures the likelihood with which the economy shifts from normalcy to disaster in a given year. The number of disaster years is defined as the number of years in the interval between peak and trough for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. The disaster probability is defined as the ratio of the number of disasters divided by the number of normalcy years. Using this set of measurement, Barro and Ursúa estimate the disaster probability to be 3.63%, the average size 22%, and the average duration 3.6 years for consumption disasters. For GDP disasters, the disaster probability is estimated to be 3.69%, the average size is 21%, and the average duration is 3.5 years.

Do macroeconomic disasters arising endogenously from the model resemble those in the data? To address this question, we first simulate the economy for 6,000 monthly periods to reach the stationary distribution. We then repeatedly simulate 1,000 artificial samples, each with 1,644 months (137 years), with the sample size matching the average sample size studied by Barro and Ursúa (2008). On each artificial sample, we time-aggregate the monthly observations of consumption and output into annual observations. We apply Barro and Ursúa's measurement on each artificial sample, and report the cross-simulation averages and the 5 and 95 percentiles for the disaster probability, size, and duration for both consumption and GDP (output) disasters.

Table 5 reports the detailed results. The average disaster size in the model is close to what we observe in the data, but the disaster probabilities and the average duration are somewhat higher than those in the data. In particular, Panel A shows that the consumption disaster probability is 5.85%, which is higher than 3.63% in the data. However, the cross-simulation standard deviation of this disaster probability is 2.29%. As such, the data moment is within the one standard deviation

**Table 5 : Moments of Macroeconomic Disasters**

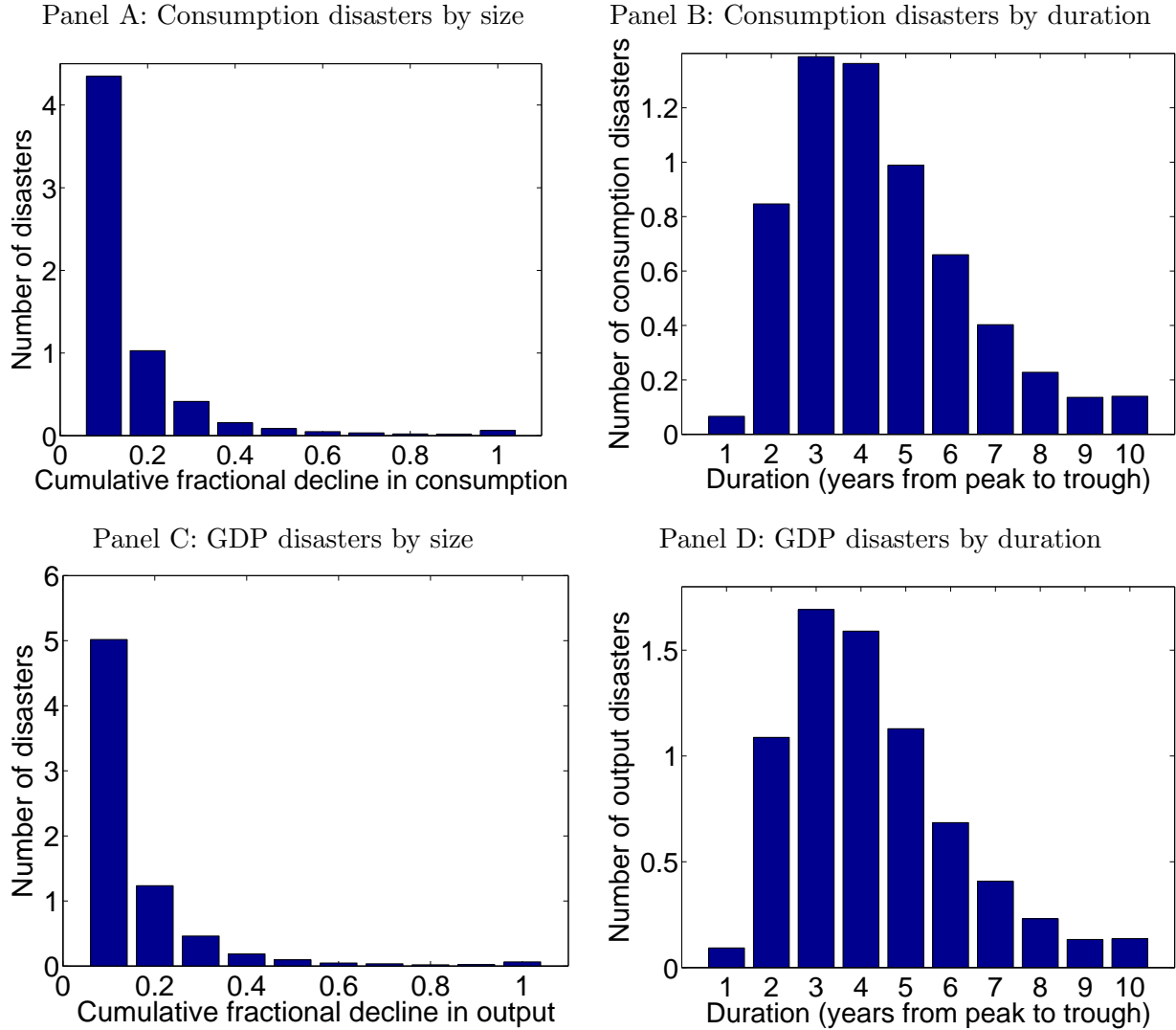
The data moments are from Barro and Ursúa (2008). The model moments are from 1,000 simulations, each with 1,644 monthly observations. We time-aggregate these monthly observations of consumption and output into 137 annual observations. On each artificial sample, we apply Barro and Ursúa’s peak-to-trough method to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is higher than its data moment. The disaster probabilities and average size are all in percent, and the average duration is in terms of years.

	Data	Model			
		Mean	5%	95%	p-value
Panel A: Consumption disasters					
Probability	3.63	5.85	2.46	9.95	0.819
Average size	22	20.75	13.05	36.09	0.311
Average duration	3.6	4.53	3.33	6.08	0.877
Panel B: GDP disasters					
Probability	3.69	6.94	3.35	11.46	0.815
Average size	21	20.63	13.65	34.39	0.349
Average duration	3.5	4.36	3.30	5.75	0.899

bound from the model’s estimate. The average duration for the consumption disasters is 4.53 years, and is longer than 3.6 years in the data. The cross-simulation standard deviation of the average duration is 0.85, meaning that the data moment is again about one standard deviation from the model’s estimate. From Panel B, the results for GDP disasters are largely similar.

Figure 6 reports the frequency distributions of consumption and GDP disasters by size and duration based on 1,000 simulations of the model. This figure is the model’s counterpart to Figures 1 and 2 in Barro and Ursúa (2008). We see that the size and duration distributions for both consumption and GDP disasters display largely similar patterns as those in the data. In particular, the size distributions seem to follow a power-law density as highlighted by Barro and Jin (2011).

Figure 6 : Distributions of Consumption and GDP Disasters by Size and Duration



## 4 What Drives the Equity Risk Premium?

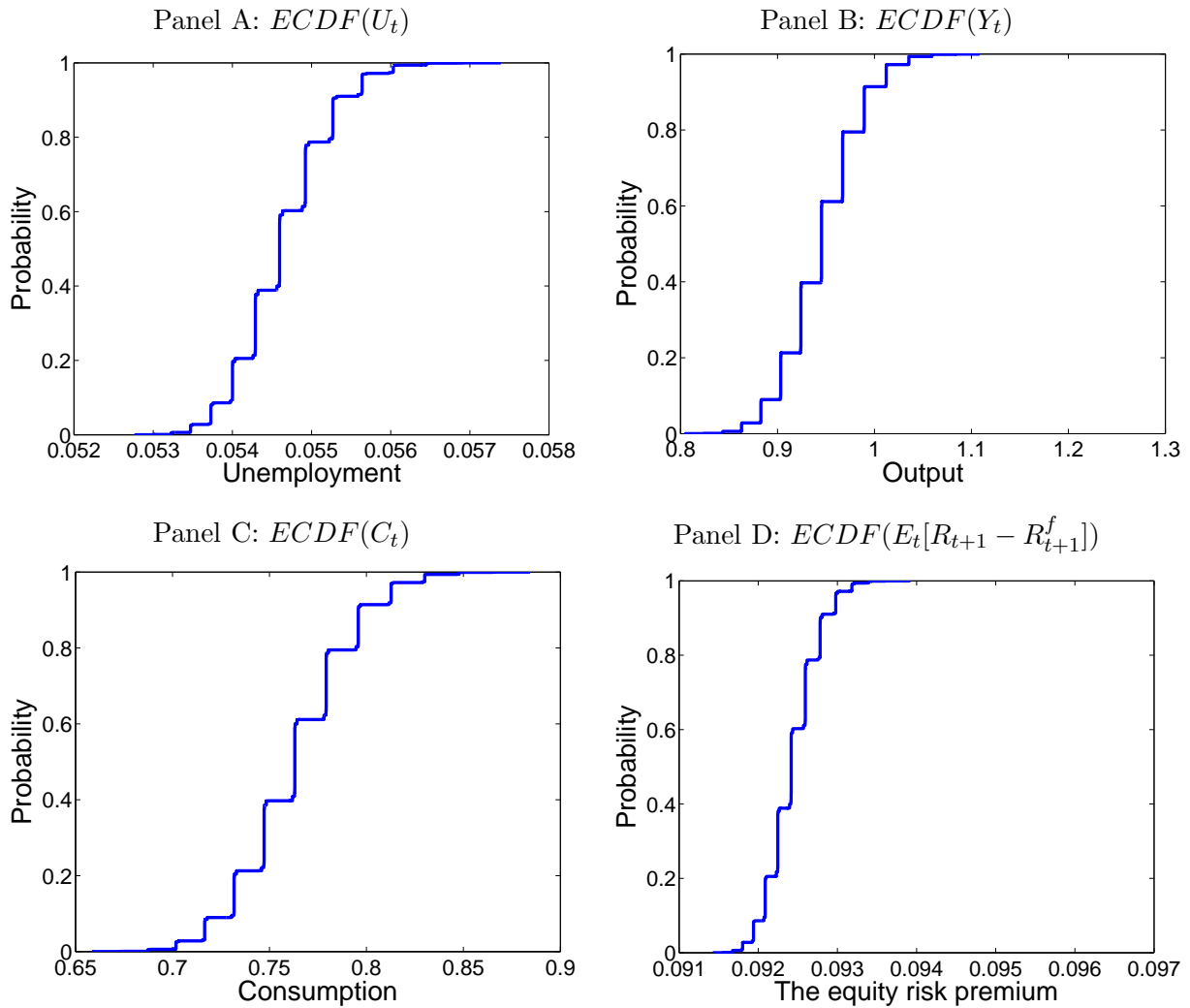
To understand the economic mechanisms underlying the equity risk premium in the model, we conduct an extensive set of comparative statics by varying the model’s key parameters.

### 4.1 The Value of Unemployment Activities

We reduce the value of unemployment benefits,  $b$ , from 0.85 in the benchmark calibration to 0.40, which is the value in Shimer (2005). All the other parameters are unchanged. Because unemployment is less valuable to workers, the average unemployment rate drops to 5.46%. A lower



**Figure 7 : Empirical Cumulative Distribution Functions, the Model with a Low Value of Unemployment Activities,  $b = 0.40$**



$b$  also means that, from equation (20), wage is more sensitive to exogenous shocks. In particular, the wage elasticity to labor productivity increases to 0.80 from 0.71 in the benchmark calibration.

From Figure 7, unlike the benchmark economy, the low- $b$  economy shows no disasters. The unemployment rate varies within a narrow range between 5.2% and 5.8%. Neither output nor consumption has a long left tail in its empirical distribution. The equity premium hovers within a narrow range around 0.09% per annum, largely time-invariant, and shows no upward spikes.

Table 6 reports the quantitative results from the low- $b$  economy. Because a lower value of  $b$  means that wage is more elastic to productivity, the marginal profits are less sensitive to shocks, and

**Table 6 : Quantitative Results from the Model with a Low Value of Unemployment Activities,  $b = 0.40$**

Results are from the model with the parameter  $b$  changed from 0.85 in the benchmark calibration to 0.40. All the other parameters remain unchanged. See Table 2's caption for the explanation of Panel A and Table 4's caption for Panel B.

Panel A: Unconditional financial moments				
	Mean	5%	95%	p-value
$\sigma^Y$	2.066	1.741	2.388	0.000
$\rho^Y(1)$	0.133	-0.036	0.294	0.000
$\rho^Y(2)$	-0.119	-0.281	0.054	0.006
$\rho^Y(3)$	-0.093	-0.282	0.088	0.782
$\rho^Y(5)$	-0.067	-0.248	0.131	0.915
$\sigma^C$	1.901	1.602	2.199	0.000
$\rho^C(1)$	0.134	-0.035	0.294	0.006
$\rho^C(2)$	-0.119	-0.282	0.055	0.036
$\rho^C(3)$	-0.093	-0.281	0.086	0.834
$\rho^C(5)$	-0.067	-0.248	0.130	0.137
$E[R - R^f]$	0.047	-0.485	0.590	0.000
$E[R^f]$	4.033	3.917	4.149	1.000
$\sigma^R$	3.519	3.223	3.827	0.000
$\sigma^{R^f}$	0.127	0.115	0.139	0.000

Panel B: Long-horizon regressions of market excess returns on labor market tightness						
	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Slope	-0.230	-0.680	-1.324	-2.510	-4.528	-6.323
$t_{NW}$	-1.047	-1.111	-1.178	-1.331	-1.687	-2.018
Adjusted $R^2$	0.268	0.780	1.497	2.833	5.140	7.265

so are employment and output. As such, the low- $b$  model produces a lower output growth volatility, 2.07% per annum, and a lower consumption growth volatility, 1.90%. The propagation mechanism is weakened as the output growth persistence and the consumption growth persistence both drop from 0.24 to 0.13. As a result of a lower amount of risk in the economy, the equity risk premium decreases from 3.67% per annum in the benchmark calibration to merely 0.05%. The market volatility also drops to 3.52%, and the  $V/U$  ratio shows no predictive power for market excess returns.

Consistent with Shimer (2005), the standard deviation of the  $V/U$  ratio in the low- $b$  model is only 0.03 versus 0.26 in the data. As such, a high value of  $b$  helps alleviate the Shimer puzzle and the equity premium puzzle simultaneously. Intuitively, by dampening the procyclical covariation

of the wage rate with productivity, a high value of  $b$  magnifies the procyclical covariation of the dividend, thereby raising the equity risk premium and the stock market volatility.

## 4.2 The Job Separation Rate

In the second experiment, we lower the job separation rate,  $s$ , from 0.05 in the benchmark calibration to 0.035. As a result, the unemployment rate drops from 12% to 7%, and the average  $V/U$  ratio increases from 0.61 to 0.79. The low- $s$  economy continues to exhibit disaster-like behavior, but it is less extreme than that in the benchmark economy. In particular, the 2.5 percentile of the unemployment rate is 4.94%, which is close to the mean of 7%. Although the 97.5 percentile of the unemployment rate is 13%, it is substantially smaller than 29% in the benchmark economy.

When we apply the Barro and Ursúa (2008) peak-to-trough measurement of disasters on the simulated data from the low- $s$  economy, we find that the economy shows less frequent and smaller disasters than the benchmark economy. In particular, the consumption disaster probability is 2.84%, and the GDP disaster probability is 3.66%. The average size of consumption disasters is 15.52%, and the average size of GDP disasters is 15.57%. However, both consumption and GDP disasters have longer average duration, 4.88 and 4.65 years, respectively, than those in the benchmark economy.

Table 7 shows that the low- $s$  economy has a smaller output growth volatility than the benchmark economy: 3.16 versus 4.95% (Panel A). The consumption growth volatility is also smaller: 2.89% versus 4.64%. The persistence of both growth rates is somewhat lower than that in the benchmark economy. Because of the lower amount of consumption risks, the equity risk premium is only 0.10% per annum in the low- $s$  economy. The risk premium is also largely time-invariant, independent of the cyclical variation of labor market tightness (Panel B).

## 4.3 The Vacancy Cost

Table 8 reports the comparative statics with the vacancy cost parameter,  $\kappa$ , changed from 0.975 in the benchmark calibration to 0.20, which is close to the value in Shimer (2005). All the other

**Table 7 : Quantitative Results from the Model with a Low Job Separation Rate,  $s = 0.035$** 

Results are from the model with the parameter  $s$  changed from 0.05 in the benchmark calibration to 0.035. All the other parameters are unchanged. See Table 2's caption for the description of Panel A and Table 4's caption for Panel B.

Panel A: Unconditional financial moments				
	Mean	5%	95%	p-value
$\sigma^Y$	3.155	2.336	4.681	0.042
$\rho^Y(1)$	0.171	-0.014	0.376	0.005
$\rho^Y(2)$	-0.127	-0.308	0.066	0.009
$\rho^Y(3)$	-0.107	-0.307	0.095	0.736
$\rho^Y(5)$	-0.069	-0.265	0.132	0.912
$\sigma^C$	2.885	2.083	4.480	0.238
$\rho^C(1)$	0.178	-0.012	0.382	0.049
$\rho^C(2)$	-0.131	-0.311	0.064	0.040
$\rho^C(3)$	-0.109	-0.311	0.090	0.798
$\rho^C(5)$	-0.070	-0.264	0.136	0.137
$E[R - R^f]$	0.103	-1.066	1.267	0.000
$E[R^f]$	4.046	3.893	4.177	1.000
$\sigma^R$	8.499	7.785	9.221	0.000
$\sigma^{R^f}$	0.764	0.509	1.226	0.006

Panel B: Long-horizon regressions of market excess returns on labor market tightness						
	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Slope	-0.482	-1.430	-2.787	-5.295	-9.688	-13.463
$t_{NW}$	-1.089	-1.152	-1.220	-1.374	-1.748	-2.081
Adjusted $R^2$	0.271	0.794	1.539	2.918	5.377	7.501

parameters are fixed. Because lower vacancy costs make hiring cheaper, the unemployment rate falls from 12.14% in the benchmark model to 5.37%. The job finding rate is on average 0.88, which is twice as high as that in the benchmark economy, 0.42. Because vacancy is more abundant, the labor market is more congested with a vacancy filling rate of only 0.22. In contrast, the vacancy filling rate is 0.73 in the benchmark economy. The average  $V/U$  ratio is 4.24 in the low- $\kappa$  economy, which is much higher than 0.61 in the benchmark economy.

Lower vacancy costs mean that the representative household is more capable of smoothing the impact of exogenous productivity shocks by varying vacancy postings. As such, the volatility of output growth falls from 4.94% to 2.18%, and the volatility of consumption growth falls from 4.64%

**Table 8 : Quantitative Results from the Model with a Low Vacancy Cost Parameter,  $\kappa = 0.20$** 

Quantitative results are from the model with the  $\kappa$  parameter changed from 0.975 in the benchmark calibration to 0.20. All the other parameters remain unchanged. See Table 2's caption for the explanation of Panel A, and Table 4's caption for Panel B.

Panel A: Unconditional financial moments					
	Mean	5%	95%	p-value	
$\sigma^Y$	2.176	1.834	2.543	0.000	
$\rho^Y(1)$	0.142	-0.030	0.310	0.000	
$\rho^Y(2)$	-0.125	-0.299	0.055	0.002	
$\rho^Y(3)$	-0.093	-0.289	0.095	0.782	
$\rho^Y(5)$	-0.067	-0.251	0.121	0.914	
$\sigma^C$	1.715	1.440	2.004	0.001	
$\rho^C(1)$	0.142	-0.030	0.312	0.008	
$\rho^C(2)$	-0.125	-0.299	0.055	0.025	
$\rho^C(3)$	-0.093	-0.286	0.095	0.845	
$\rho^C(5)$	-0.067	-0.252	0.121	0.116	
$E[R - R^f]$	-0.688	-3.468	1.691	0.000	
$E[R^f]$	4.026	3.925	4.123	1.000	
$\sigma^R$	15.802	13.697	18.348	0.990	
$\sigma^{R^f}$	0.213	0.147	0.323	0.001	

Panel B: Long-horizon regressions of market excess returns on labor market tightness						
	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Slope	-0.174	-0.512	-0.991	-1.905	-3.515	-4.877
$t_{NW}$	-0.840	-0.884	-0.938	-1.086	-1.418	-1.703
Adjusted $R^2$	0.252	0.742	1.404	2.662	4.863	6.705

to 1.72% per annum. The autocorrelations of both fall from 0.24 to 0.14. As a result of consumption smoothing, the equity premium is even slightly negative, -0.69% per annum. The equity premium is also largely time-invariant, and is not predictable by labor market tightness. Also, because vacancy is more responsive to shocks, so is the shadow value of an additional unit of labor,  $\mu_t$ , from period to period. From equation (15), the sensitivity of  $\mu_t$  translates to a high market volatility, 15.80% per annum. This volatility is almost twice as large as the market volatility in the benchmark economy.

#### 4.4 The Workers' Bargaining Weight

Table 9 reports the results from changing the workers' bargaining weight,  $\eta$ , from 0.10 in the benchmark economy to 0.25. A higher  $\eta$  makes the wage more procyclical and profits less procyclical.

**Table 9 : Quantitative Results from the Model with a High Workers' Bargaining Weight,**  
 $\eta = 0.25$

Quantitative results are from the model with the  $\eta$  parameter changed from 0.10 in the benchmark calibration to 0.25. All the other parameters remain unchanged. See Table 2's caption for the explanation of Panel A, and Table 4's caption for Panel B.

Panel A: Unconditional financial moments				
	Mean	5%	95%	p-value
$\sigma^Y$	7.637	4.710	13.491	0.907
$\rho^Y(1)$	0.423	0.200	0.676	0.212
$\rho^Y(2)$	-0.025	-0.257	0.293	0.119
$\rho^Y(3)$	-0.129	-0.350	0.118	0.616
$\rho^Y(5)$	-0.127	-0.348	0.103	0.765
$\sigma^C$	7.557	4.592	13.443	1.000
$\rho^C(1)$	0.433	0.207	0.680	0.625
$\rho^C(2)$	-0.024	-0.259	0.295	0.236
$\rho^C(3)$	-0.130	-0.350	0.121	0.689
$\rho^C(5)$	-0.129	-0.353	0.100	0.086
$E[R - R^f]$	0.880	0.581	1.220	0.000
$E[R^f]$	3.537	3.059	3.973	1.000
$\sigma^R$	3.741	3.285	4.262	0.000
$\sigma^{R^f}$	1.846	1.210	2.979	0.389

Panel B: Long-horizon regressions of market excess returns on labor market tightness						
	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Slope	-0.715	-2.096	-4.053	-7.580	-13.472	-18.337
$t_{NW}$	-1.732	-1.806	-1.899	-2.117	-2.687	-3.191
Adjusted $R^2$	0.490	1.429	2.749	5.148	9.247	12.686

This effect weakens the operating leverage mechanism, and lowers the equity risk premium. Also, the wage rate eats up a higher portion of profits, reducing the firm's incentives to hire. As such, the unemployment rate goes up to 23.33% from 12.14% in the benchmark economy.

The high- $\eta$  economy shows more volatile and more persistent consumption growth and output growth than the benchmark economy. Because a higher  $\eta$  makes wages more procyclical and profits less procyclical, vacancies are less responsive to shocks. As such, unemployment (and employment) become more responsive to shocks, making output more responsive to shocks as well. From equation (21), consumption must also absorb more shocks to become more volatile and persistent.

## 4.5 Risk Aversion

In the final experiment, we vary risk aversion,  $\gamma$ , from ten in the benchmark economy to zero (risk neutrality). All the other parameters are unchanged. The elasticity of intertemporal substitution,  $\psi$ , is pegged at 1.5 to keep the interest rate volatility low. Consistent with Tallarini (2000), risk aversion only affects asset pricing moments, while leaving business cycle moments largely unchanged.

Table 10 reports the quantitative results for the risk-neutral economy. The equity premium drops from 3.67% in the benchmark economy to  $-0.33\%$ . The equity premium is largely time-invariant. However, the volatility and persistence of output growth and consumption growth are quantitatively close to the benchmark economy. Further, the labor market moments hardly change. In particular, the standard deviation of the  $V/U$  ratio and the  $V-U$  correlation are 0.15 and  $-0.52$ , which are close to 0.16 and  $-0.51$  in the benchmark calibration, respectively.

## 5 Conclusion

We study aggregate asset pricing with production by embedding the standard Diamond-Mortensen-Pissarides search model of the labor market into a dynamic stochastic general equilibrium economy. We find that search frictions in the labor market help explain the equity premium in the financial market. With reasonable parameter values, the model reproduces a sizeable equity risk premium with a low interest rate volatility. Also, the equity premium is countercyclical, and is forecastable by labor market tightness, a prediction that we confirm in the data. Intuitively, search frictions, especially a small labor surplus, combined with large job destruction flows, create endogenously rare macroeconomic disasters as in Rietz (1988) and Barro (2006) in the economy. In addition, by strengthening the model's propagation mechanism, search frictions increase the persistence in output growth and consumption growth, giving rise endogenously to long run risks as in Bansal and Yaron (2004). Both disaster risks and long run risks help explain the equity premium.

**Table 10 : Quantitative Results from the Model with Low Risk Aversion,  $\gamma = 0$**

Results are from the model with risk aversion  $\gamma = 0$ . All the other parameters remain unchanged. See Table 2's caption for the explanation of Panel A, and Table 4's caption for Panel B.

Panel A: Unconditional financial moments				
	Mean	5%	95%	p-value
$\sigma^Y$	4.752	2.916	9.631	0.279
$\rho^Y(1)$	0.227	0.000	0.546	0.052
$\rho^Y(2)$	-0.129	-0.330	0.119	0.031
$\rho^Y(3)$	-0.125	-0.334	0.091	0.658
$\rho^Y(5)$	-0.082	-0.302	0.129	0.856
$\sigma^C$	4.445	2.584	9.411	0.768
$\rho^C(1)$	0.233	0.003	0.554	0.139
$\rho^C(2)$	-0.134	-0.337	0.119	0.076
$\rho^C(3)$	-0.128	-0.337	0.092	0.736
$\rho^C(5)$	-0.084	-0.307	0.134	0.137
$E[R - R^f]$	-0.326	-1.389	0.720	0.000
$E[R^f]$	3.975	3.712	4.200	1.000
$\sigma^R$	7.792	7.096	8.454	0.000
$\sigma^{R^f}$	1.325	0.839	2.355	0.107

Panel B: Long-horizon regressions of market excess returns on labor market tightness						
	Forecast horizon ( $H$ ) in months					
	1	3	6	12	24	36
Slope	-0.478	-1.410	-2.752	-5.226	-9.559	-13.238
$t_{NW}$	-1.137	-1.193	-1.260	-1.415	-1.779	-2.095
Adjusted $R^2$	0.275	0.800	1.552	2.936	5.479	7.682



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## A The Stock Return Equation

We prove equation (15) following an analogous proof in Liu, Whited, and Zhang (2009) in the context of the  $q$ -theory of investment. Rewrite the equity value maximization problem as:

$$S_t = \max_{\{V_{t+\Delta t}, N_{t+\Delta t+1}\}} E_t \left[ \sum_{\Delta t=0}^{\infty} M_{t+\Delta t} \begin{bmatrix} X_{t+\Delta t} N_{t+\Delta t} - W_{t+\Delta t} N_{t+\Delta t} - \kappa V_{t+\Delta t} \\ -\mu_{t+\Delta t} [N_{t+\Delta t+1} - (1-s)N_{t+\Delta t}] \\ -V_{t+\Delta t} q(\theta_{t+\Delta t}) + \lambda_{t+\Delta t} q(\theta_{t+\Delta t}) V_{t+\Delta t} \end{bmatrix} \right], \quad (\text{A.1})$$

in which  $\mu_t$  is the Lagrange multiplier on the employment accumulation equation, and  $\lambda_t$  is the Lagrange multiplier on the irreversibility constraint on job creation. The first order conditions are equations (10) and (11), and the Kuhn-Tucker condition is equation (14).

Define dividends as  $D_t = X_t N_t - W_t N_t - \kappa V_t$  and the ex-dividend equity value as  $P_t = S_t - D_t$ . Expanding  $S_t$  yields:

$$\begin{aligned} P_t + X_t N_t - W_t N_t - \kappa V_t = S_t &= X_t N_t - W_t N_t - \kappa V_t - \mu_t [N_{t+1} - (1-s)N_t - V_t q(\theta_t)] + \lambda_t q(\theta_t) V_t \\ &+ E_t M_{t+1} [X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa V_{t+1} - \mu_{t+1} [N_{t+2} - (1-s)N_{t+1} - V_{t+1} q(\theta_{t+1})] \\ &+ \lambda_{t+1} q(\theta_{t+1}) V_{t+1}] + \dots \end{aligned} \quad (\text{A.2})$$

Recursively substituting equations (10) and (11) yields:  $P_t + X_t N_t - W_t N_t - \kappa V_t = X_t N_t - W_t N_t + \mu_t (1-s)N_t$ . Using equation (10) to simplify further:  $P_t = \kappa V_t + \mu_t (1-s)N_t = \mu_t [(1-s)N_t + q(\theta_t) V_t] + \lambda_t q(\theta_t) V_t = \mu_t N_{t+1}$ , in which the last equality follows from the Kuhn-Tucker condition (14).

To show equation (15), we expand the stock returns:

$$\begin{aligned} R_{t+1} &= \frac{S_{t+1}}{S_t - D_t} = \frac{\mu_{t+1} N_{t+2} + X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} - \kappa \frac{V_{t+1}}{N_{t+1}} + \mu_{t+1} \left[ (1-s) + q(\theta_{t+1}) \frac{V_{t+1}}{N_{t+1}} \right]}{\mu_t} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t} + \frac{1}{\mu_t N_{t+1}} [\mu_{t+1} q(\theta_{t+1}) V_{t+1} - \kappa V_{t+1}] \\ &= \frac{X_{t+1} - W_{t+1} + (1-s)\mu_{t+1}}{\mu_t}, \end{aligned} \quad (\text{A.3})$$

in which the last equality follows because  $\mu_{t+1} q(\theta_{t+1}) V_{t+1} - \kappa V_{t+1} = -\lambda_{t+1} q(\theta_{t+1}) V_{t+1} = 0$  from the Kuhn-Tucker condition. ■

## B Wage Determination under Nash Bargaining

Let  $0 < \eta < 1$  denote the relative bargaining weight of the worker,  $J_{N_t}$  the marginal value of an employed worker to the representative family,  $J_{U_t}$  the marginal value of an unemployed worker to the

representative family,  $\phi_t$  the marginal utility of the representative family,  $S_{Nt}$  the marginal value of an employed worker to the representative firm, and  $S_{Vt}$  the marginal value of an unemployed worker to the representative firm. Let  $\Lambda_t \equiv (J_{Nt} - J_{Ut})/\phi_t + S_{Nt} - S_{Vt}$  be the total surplus from the Nash bargain. The wage equation (20) is determined via the Nash worker-firm bargain:

$$\max_{\{W_t\}} \left( \frac{J_{Nt} - J_{Ut}}{\phi_t} \right)^\eta (S_{Nt} - S_{Vt})^{1-\eta}, \quad (\text{B.1})$$

The outcome of maximizing equation (B.1) is the surplus-sharing rule:

$$\frac{J_{Nt} - J_{Ut}}{\phi_t} = \eta \Lambda_t = \eta \left( \frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt} \right). \quad (\text{B.2})$$

As such, the worker receives a fraction of  $\eta$  of the total surplus from the wage bargain. In what follows, we derive the wage equation (20) from the sharing rule in equation (B.2).

## B.1 Workers

Let  $\phi_t$  denote the Lagrange multiplier for the household's budget constraint (17). The household's maximization problem is given by:

$$J_t = \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}} - \phi_t \left( \frac{\Pi_{t+1}}{R_{t+1}^\Pi} - \Pi_t + C_t - W_t N_t - U_t b + T_t \right), \quad (\text{B.3})$$

The first-order condition for consumption yields:

$$\phi_t = (1 - \beta)C_t^{-\frac{1}{\psi}} \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}-1}, \quad (\text{B.4})$$

which gives the marginal utility of consumption.

Recalling  $N_{t+1} = (1 - s)N_t + f(\theta_t)U_t$  and  $U_{t+1} = sN_t + (1 - f(\theta_t))U_t$ , we differentiate  $J_t$  in equation (B.3) with respect to  $N_t$ :

$$\begin{aligned} J_{Nt} &= \phi_t W_t + \frac{1}{1 - \frac{1}{\psi}} \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times \frac{1 - \frac{1}{\psi}}{1 - \gamma} \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t \left[ (1 - \gamma) J_{t+1}^{-\gamma} [(1 - s)J_{Nt+1} + sJ_{Ut+1}] \right]. \quad (\text{B.5}) \end{aligned}$$

Dividing both sides by  $\phi_t$ :

$$\frac{J_{Nt}}{\phi_t} = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[ \frac{1}{\left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t \left[ J_{t+1}^{-\gamma} [(1-s)J_{Nt+1} + sJ_{Ut+1}] \right]. \quad (\text{B.6})$$

Dividing and multiplying by  $\phi_{t+1}$ :

$$\begin{aligned} \frac{J_{Nt}}{\phi_t} &= W_t + E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left[ \frac{J_{t+1}}{\left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} \left[ (1-s) \frac{J_{Nt+1}}{\phi_{t+1}} + s \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right] \\ &= W_t + E_t \left[ M_{t+1} \left[ (1-s) \frac{J_{Nt+1}}{\phi_{t+1}} + s \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right]. \end{aligned} \quad (\text{B.7})$$

Similarly, differentiating  $J_t$  in equation (B.3) with respect to  $U_t$  yields:

$$\begin{aligned} J_{Ut} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}} \left[ (1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1-1/\psi}{1-1/\psi}-1} \\ &\quad \times \frac{1-\frac{1}{\psi}}{1-\gamma} \beta \left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t \left[ (1-\gamma) J_{t+1}^{-\gamma} [f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}] \right]. \end{aligned} \quad (\text{B.8})$$

Dividing both sides by  $\phi_t$ :

$$\frac{J_{Ut}}{\phi_t} = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[ \frac{1}{\left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t \left[ J_{t+1}^{-\gamma} [f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}] \right]. \quad (\text{B.9})$$

Dividing and multiplying by  $\phi_{t+1}$ :

$$\begin{aligned} \frac{J_{Ut}}{\phi_t} &= b + E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left[ \frac{J_{t+1}}{\left[ E_t \left( J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} \left[ f(\theta_t) \frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t)) \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right] \\ &= b + E_t \left[ M_{t+1} \left[ f(\theta_t) \frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t)) \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right]. \end{aligned} \quad (\text{B.10})$$

## B.2 The Firm

We start by rewriting the infinite-horizon value-maximization problem of the firm recursively as:

$$S_t = X_t N_t - W_t N_t - \kappa V_t + \lambda_t q(\theta_t) V_t + E_t [M_{t+1} S_{t+1}], \quad (\text{B.11})$$

subject to  $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$ . The first-order condition with respect to  $V_t$  says:

$$S_{Vt} = -\kappa + \lambda_t q(\theta_t) + E_t[M_{t+1}S_{Nt+1}q(\theta_t)] = 0 \quad (\text{B.12})$$

Equivalently,

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t[M_{t+1}S_{Nt+1}] \quad (\text{B.13})$$

In addition, differentiating  $S_t$  with respect to  $N_t$  yields:

$$S_{Nt} = X_t - W_t + E_t[M_{t+1}(1-s)S_{Nt+1}]. \quad (\text{B.14})$$

Combining the last two equations yields the intertemporal job creation condition in equation (12).

### B.3 The Wage Equation

From equations (B.7), (B.10), and (B.14), the total surplus of the worker-firm relationship is:

$$\begin{aligned} \Lambda_t &= W_t + E_t \left[ M_{t+1} \left[ (1-s) \frac{J_{Nt+1}}{\phi_{t+1}} + s \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right] \\ &\quad - b - E_t \left[ M_{t+1} \left[ f(\theta_t) \frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t)) \frac{J_{Ut+1}}{\phi_{t+1}} \right] \right] \\ &\quad + X_t - W_t + E_t[M_{t+1}(1-s)S_{Nt+1}] \\ &= X_t - b + (1-s)E_t \left[ M_{t+1} \left( \frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} + S_{Nt+1} \right) \right] - f(\theta_t)E_t \left[ M_{t+1} \frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} \right] \\ &= X_t - b + (1-s)E_t[M_{t+1}\Lambda_{t+1}] - \eta f(\theta_t)E_t[M_{t+1}\Lambda_{t+1}], \end{aligned} \quad (\text{B.15})$$

in which the last equality follows from the definition of  $\Lambda_t$  and the surplus sharing rule (B.2).

The surplus sharing rule implies  $S_{Nt} = (1-\eta)\Lambda_t$ , which, combined with equation (B.14), yields:

$$(1-\eta)\Lambda_t = X_t - W_t + (1-\eta)(1-s)E_t[M_{t+1}\Lambda_{t+1}]. \quad (\text{B.16})$$

Combining equations (B.15) and (B.16) yields:

$$\begin{aligned} X_t - W_t + (1-\eta)(1-s)E_t[M_{t+1}\Lambda_{t+1}] &= (1-\eta)(X_t - b) + (1-\eta)(1-s)E_t[M_{t+1}\Lambda_{t+1}] \\ &\quad - (1-\eta)\eta f(\theta_t)E_t[M_{t+1}\Lambda_{t+1}] \\ X_t - W_t &= (1-\eta)(X_t - b) - (1-\eta)\eta f(\theta_t)E_t[M_{t+1}\Lambda_{t+1}] \\ W_t &= \eta X_t + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t[M_{t+1}\Lambda_{t+1}] \end{aligned}$$

Using equations (B.2) and (B.13) to simplify further:

$$W_t = \eta X_t + (1 - \eta)b + \eta f(\theta_t) E_t [M_{t+1} S_{N_{t+1}}] \quad (\text{B.17})$$

$$W_t = \eta X_t + (1 - \eta)b + \eta f(\theta_t) \left( \frac{\kappa}{q(\theta_t)} - \lambda_t \right) \quad (\text{B.18})$$

Using the Kuhn-Tucker condition, when  $V_t > 0$ , then  $\lambda_t = 0$ , and equation (B.18) reduces to the wage equation (20) because  $f(\theta_t) = \theta_t q(\theta_t)$ . On the other hand, when the irreversibility constraint is binding,  $\lambda_t > 0$ , but  $V_t = 0$  means  $\theta_t = 0$  and  $f(\theta_t) = 0$ . Equation (B.18) reduces to  $W_t = \eta X_t + (1 - \eta)b$ . Because  $\theta_t = 0$ , the wage equation (20) continues to hold.

## C Nonlinear Solution: Further Details

In the numerical implementation, the two functional equations should be expressed only in terms of state variables  $N_t$  and  $x_t$ . To this end, we perform the following set of substitutions:

$$N_{t+1} = (1 - s)N_t + \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}; \quad (\text{C.1})$$

$$U_t = 1 - N_t; \quad (\text{C.2})$$

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}; \quad (\text{C.3})$$

$$C(N_t, x_t) = \exp(x_t) N_t - \kappa V(N_t, x_t); \quad (\text{C.4})$$

$$q(\theta_t) = \left[ 1 + \left( \frac{V(N_t, x_t)}{1 - N_t} \right)^\iota \right]^{-\frac{1}{\iota}}; \quad (\text{C.5})$$

$$M_{t+1} = \beta \left[ \frac{C(N_{t+1}, x_{t+1})}{C(N_t, x_t)} \right]^{-\frac{1}{\psi}} \left[ \frac{J(N_{t+1}, x_{t+1})}{E_t [J(N_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi} - \gamma}; \quad (\text{C.6})$$

and

$$W_t = \eta \left[ \exp(x_t) + \kappa \frac{V(N_t, x_t)}{1 - N_t} \right] + (1 - \eta)b. \quad (\text{C.7})$$

## D Additional Properties of the Model

Figure D.1 reports additional properties of the model's solution not shown in Section 2.4. The indirect utility function,  $J_t$ , is increasing in both employment and productivity (Panel A). The multiplier on the employment accumulation equation (4),  $\mu_t$ , is identical to the conditional expectation,  $\mathcal{E}_t$  (Panel B). Not surprisingly, optimal consumption,  $C_t$ , is weakly procyclical, and is decreasing in employment (Panel C). Sensibly, the ex-dividend stock price,  $P_t$ , is procyclical, and is increasing in employment (Panel D). The job finding rate,  $f(\theta_t)$ , is procyclical, and is increasing in employment (Panel E). Finally, the wage rate is also procyclical, and is increasing in employment (Panel F).



**Figure D.1 : Additional Properties of the Model: Indirect Utility, the Multiplier on Employment Accumulation, Consumption, Stock Price, the Job Finding Rate, and Wage**

We plot the following variables on the two-dimensional  $N_t$ - $x_t$  grid:  $J_t$  is the indirect utility in equation (22);  $\mu_t$  is the multiplier on the employment accumulation equation (4);  $C_t$  is consumption;  $P_t = \mu_t N_{t+1}$  is the ex-dividend stock price;  $f(\theta_t)$  is the job finding rate; and  $W_t$  is the wage rate.

