Size Premium Waves

Bernard Herskovic[*]   Thilo Kind[†]   Howard Kung[‡]

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Abstract

This paper examines the link between microeconomic uncertainty and the size premium across different frequencies in an investment model with heterogeneous firms. We document that the observed time-varying dispersion in firm-specific productivity can account for a large size premium in the 1960's and 1970's, the disappearance in the 1980's and 1990's, and reemergence in the 2000's. Periods with a large (small) size premium coincide with high (low) microeconomic uncertainty. During episodes of high productivity dispersion, small firms increase their exposure to macroeconomic risks. Our model can also explain the strong positive low-frequency co-movement between size and value factors, but a negative relation with the market factor.

Keywords: Size premium, Cross-section of returns, Uncertainty, TFP, Investment

1   Introduction

The relation between firm size and expected stock returns has varied significantly over time in waves. Banz (1981) documented a size premium whereby firms with small market capitalizations earn higher expected returns than large ones before 1975, and that this size effect cannot be explained by market betas. The size effect subsequently vanished starting in the early 1980s to the late 1990s, before reemerging after 2000 (see Table 1). We also observe that measures of microeconomic uncertainty, such as the cross-sectional dispersion in plant- and firm-level total factor productivity (TFP), sales, and payouts, exhibit similar low-frequency patterns as the size premium. Figure 1 illustrates that microeconomic uncertainty is strongly positively correlated with the size premium. In this paper, we demonstrate how persistent variation in microeconomic uncertainty can potentially rationalize the observed size premium waves.

To this end, we build a dynamic partial equilibrium production model with heterogeneous firms. The model has several distinguishing features. First, firms are subject to persistent idiosyncratic and aggregate TFP shocks with time-varying second moments. The second moment shocks to the idiosyncratic component capture time-varying *cross-sectional* dispersion in idiosyncratic productivity (microeconomic uncertainty) while the second moment shocks to the aggregate component capture fluctuations in macroeconomic uncertainty. Second, firms face quadratic adjustment costs and operating costs. Third, the representative household has recursive utility defined over aggregate streams of consumption.

We find that our calibrated model produces a realistic size premium and captures the salient dynamics of the size premium across different frequencies. Namely, the model generates a counter-cyclical size premium and reproduces the low-frequency wave patterns, including a large spread during 1960-1980, a disappearance between 1980-2000, and resurgence post-2000. The mean-reverting idiosyncratic TFP shocks helps to generate a negative relation between firm market capitalization and expected returns in the stationary distribution. Small firms are those that have received a recent history of negative idiosyncratic shocks. Due to mean reversion, the shorter-term payouts of small firms therefore constitute a smaller share of aggregate payouts relative to their longer-term payouts. With a similar logic, the payout shares of large firms have the opposite pattern. Conse-

quently, small firms are more exposed to aggregate long-run risks than large firms, which gives rise to a quantitatively significant size premium.

The low-frequency fluctuations of the size premium in the model are driven by the persistent volatility process for idiosyncratic TFP shocks. When TFP dispersion is high, small firms are subjected to a larger history of negative idiosyncratic shocks that increases their exposure to long-run risks relative to periods with low TFP dispersion. As a result, the size premium is larger during periods of higher TFP dispersion. In the data, we find a very strong association between TFP dispersion and the size premium at low frequencies, consistent with the model predictions. Calibrating the idiosyncratic volatility process to our empirical measure, we show that our model can provide a quantitatively relevant account of the observed size premium waves.

The model also generates significant equity and value premia, inline with the observed magnitudes in the data. Persistent shocks to aggregate productivity growth are a source of long-run risks (e.g. Bansal and Yaron (2004)) that help to generate a sizable equity premium when coupled with recursive preferences. Persistent second moment shocks to aggregate productivity growth generate a countercyclical equity premium.

A value premium arises due to the combination of the asymmetric capital adjustment costs and operating costs, in a similar spirit as Zhang (2005). Firms with high book-to-market ratios have large stocks of capital, but have experienced a recent history of negative idiosyncratic shocks. Therefore, such firms have strong incentives to disinvest due to the low marginal product of capital and high operating costs, but the presence of capital adjustment costs prevents them from selling off their unproductive capital rapidly, which exposes high book-to-market (value) firms more to adverse aggregate shocks than low book-to-market (growth) firms. In particular, discouraging aggressive disinvestment policies prevents firms with large capital stocks from increasing payouts financed through capital sales in response to negative idiosyncratic shocks. The operating costs that are proportional to the capital stock of the firm reduce the funds available for payouts, especially for large firms. Therefore, these investment frictions imply that high book-to-market firms have low payout shares today, but higher payout shares at longer horizons due to mean reversion. Therefore, value firms are more exposed to long-run risks than growth firms, thereby generating a sizable value premium. The low-frequency fluctuations of the value premium are driven by the persistent

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idiosyncratic volatility process.

The model can also jointly explain the strong positive comovement between size and value premia at low frequencies, but a negative relation between size and value premia with the aggregate market premium. We define low-frequency fluctuations as frequencies between 20 and 50 years. In the model, the low-frequency dynamics for the size and value premia are driven by the stochastic process for microeconomic uncertainty, while the low-frequency dynamics of the equity premium are driven by the stochastic process for macroeconomic uncertainty. In the data, there is a negative association between measures of microeconomic and macroeconomic uncertainty at low-frequencies. Calibrating the correlation between the volatility processes to be consistent with the data, we therefore obtain negative relation of the size and value premia with the market premium, consistent with the observed patterns in the data.

1.1 Related Literature

Our paper relates to investment-based models studying the cross-section of stock returns (e.g., Berk, Green, and Naik (1999), Babenko, Boguth, and Tserlukevich (2016) Carlson, Fisher, and Giammarino (2004), Zhang (2005), Kogan and Papanikolaou (2014), Gomes and Schmid (2010), Belo and Lin (2011), Belo, Lin, and Vitorino (2014), Clementi and Palazzo (2015), and Ai and Kiku (2015)). We also build on general equilibrium production-based models studying the cross-section of returns (e.g., Gomes, Kogan, and Zhang (2003), Ai, Croce, and Li (2012), Favilukis and Lin (2013), Favilukis and Lin (2015), and Bai, Hou, Kung, Li, and Zhang (2018)). Finally, we relate to the macroeconomics literature studying the business cycle effects of shocks to the dispersion in firm-level productivity (e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Bachmann and Bayer (2014), and Crouzet, Mehrotra, et al. (2017)).

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2 Stylized Facts

We document four stylized facts regarding risk premia and uncertainty dynamics at low frequencies. We isolate the low-frequency component by using a bandpass filter and selecting the frequency

between 20 and 50 years.

- *Stylized fact 1.* The size premium and microeconomic uncertainty (i.e., TFP dispersion) exhibit strong positive comovement at low frequencies (Figure 1). The correlation is 0.81. This pattern is robust to different measures of size (Figure 5).

- *Stylized fact 2.* The size an value premia are strongly positively related at low frequencies (i.e., correlation of 0.66), but they are both negatively related with the equity premium at low frequencies (correlation between the size premium and the equity premium is -0.62 and the correlation between the value premium and the equity premium is -0.50). Figure 2 provides a visual depiction of these relations.

- *Stylized fact 3.* The equity premium is strongly correlated with macroeconomic uncertainty, as measured by the realized volatility of consumption growth, output growth, and TFP (Figure 4), but negatively related to microeconomic uncertainty at low frequencies. The correlation between the equity premium and macroeconomic uncertainty is 0.76, while the correlation between the equity premium and microeconomic uncertainty is -0.64.

- *Stylized fact 4.* Microeconomic and macroeconomic uncertainty are negatively related at low frequencies (Figure 4). The correlation is -0.72.

3 Model

This section presents the partial equilibrium model with heterogeneous firms. On the household side, a representative agent has recursive preferences over aggregate consumption, which evolves exogenously. The production-side consists of firms which use capital as input to produce a homogeneous good. Firm-level productivity consists of an aggregate and an idiosyncratic component which are both subject to time-varying volatility. In addition, capital investments entail asymmetric quadratic adjustment costs and operating costs that are proportional to firm size and aggregate productivity growth.

We use this model to provide a quantitative explanation for the stylized facts documented in the previous section.

3.1 Household

The representative agent has recursive utility, U_t, over aggregate consumption C_t:

$$U_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

where β is the coefficient of time discount factor, ψ is the coefficient of intertemporal elasticity of substitution, and γ is the coefficient of relative risk aversion. The stochastic discount factor, $M_{t,t+1}$, is given by:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{2}$$

Consumption growth evolves exogenously as in Bansal and Yaron (2004):

$$x_{c,t} = \bar{x} + x_t + \sigma_{x,t-1} \varepsilon_{c,t}, \quad \varepsilon_{c,t} \sim \text{iid } \mathcal{N}(0,1) \tag{3}$$

$$x_t = \rho_x x_{t-1} + \sigma_\varphi \sigma_{x,t-1} \varepsilon_{x,t}, \quad \varepsilon_{x,t} \sim \text{iid } \mathcal{N}(0,1), \tag{4}$$

where $x_{c,t} \equiv \log(C_t/C_{t-1})$ is the log consumption growth. The conditional volatility of the exogenous process $\sigma_{x,t}$ follows a two-state Markov chain which is characterized by it's state value S_x and the transition matrix P_x:

$$S_x = \left\{ \sigma_x^L, \sigma_x^H \right\}, \qquad P_x = \begin{pmatrix} p_{HH}^x & 1 - p_{LL}^x \\ 1 - p_{HH}^x & p_{LL}^x \end{pmatrix} \tag{5}$$

3.2 Firms

A continuum of competitive firms produce a homogeneous good. Given aggregate productivity $X_{y,t}$, firm-specific productivity $Z_{i,t}$, and capital $K_{i,t}$, operating profits of firm i are given by the following technology:

$$Y_{i,t} = (X_{y,t} Z_{i,t})^{1-\alpha} K_{i,t}^\alpha - f K_{i,t} - \bar{f} X_{y,t-1}, \tag{6}$$

where $f > 0$ represent the proportional operating costs of production. $\bar{f}$ is the non-proportional fixed costs scaled by the level of lagged aggregate productivity which ensures a non-zero effect along the growth-path. Further, α is the parameter corresponding to the output elasticity of capital. The log aggregate productivity growth $x_{y,t} \equiv \log\left(X_{y,t}/X_{y,t-1}\right)$ and the firm-specific idiosyncratic productivity $z_{i,t} \equiv \log\left(Z_{i,t}\right)$ are modeled via:

$$z_{i,t} = (1 - \rho)\,\mu_z + \rho_z z_{i,t-1} + \sigma_{z,t-1}\varepsilon_{i,t} \tag{7}$$

$$x_{y,t} = \bar{x} + \phi x_t, \tag{8}$$

where $\phi > 0$, $\varepsilon_{i,t} \sim$ i.i.d. $\mathcal{N}(0,1)$, and $corr(\varepsilon_{i,t}, \varepsilon_{j,t}) = 0\,\forall\,i \neq j$. Here, $\sigma_{z,t}$ is the time-varying dispersion in the idiosyncratic productivity component. The conditional volatility of log firm-specific productivity follows a two-state Markov Chain with it's state value S_z and it's transition matrix P_z:

$$S_z = \left\{\sigma_z^L, \sigma_z^H\right\}, \qquad P_z = \begin{pmatrix} p_{HH}^z & 1 - p_{LL}^z \\ 1 - p_{HH}^z & p_{LL}^z \end{pmatrix}. \tag{9}$$

To capture correlation between $\sigma_{x,t}$ and $\sigma_{z,t}$, we model the two processes jointly as a four-state Markov Chain. Further, each firm invests $I_{i,t}$ and accumulates capital according to

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t} \tag{10}$$

where δ is the rate of depreciation. In addition, capital investment is subject to an asymmetric quadratic:

$$H\left(\frac{I_{i,t}}{K_{i,t}}\right) \equiv \frac{\theta_t}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}, \quad \text{where } \theta_t = \begin{cases} \theta^+ & \text{for } I_{i,t} \geqslant 0 \\ \theta^- & \text{for } I_{i,t} < 0, \end{cases} \tag{11}$$

for $\theta^- > \theta^+ > 0$. Then, each firm maximizes the cum-dividend market equity $V_{i,t}$, given the exogenous stochastic discount factor $M_{t,t+1}$. In particular, firm i optimizes over investment $I_{i,t}$ and

makes an optimal exit decision $\chi_{i,t}$:

$$V_{i,t} = \max_{\chi_{i,t}} \left\{ \max_{I_{i,t}} D_{i,t} + \mathbb{E}_t \left[M_{t,t+1} V_{i,t+1} \right], \kappa X_{y,t-1} \right\} \tag{12}$$

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t} \tag{13}$$

$$D_{i,t} = (X_{y,t} Z_{i,t})^{1-\alpha} K_{i,t}^{\alpha} - f K_{i,t} - \bar{f} X_{y,t-1} - H_{i,t} - I_{i,t} \tag{14}$$

When the market value of equity is equal to the lower threshold κ, firm i exits the economy. In this case, the firm is liquidated and reorganized at the beginning of period t. The old firm is replaced by an entry of a new firm with the old physical capital stock $K_{i,t}$. The new firm-specific idiosyncratic productivity $z_{i,t}$ is drawn from the it's unconditional distribution. In the event of liquidation the associated return of firm i between period $t - 1$ and t is set to the average delisting return $\bar{R}$.

3.3   Stationary Equilibrium

To solve for the competitive equilibrium along the balanced growth path, we define the stationary variables $\hat{U}_t \equiv U_t/C_t$, $\hat{K}_{i,t} \equiv K_{i,t}/X_{y,t-1}$, $\hat{V}_{i,t} \equiv V_{i,t}/X_{y,t-1}$, $\hat{Y}_{i,t} \equiv Y_{i,t}/X_{y,t-1}$, $\hat{I}_{i,t} \equiv I_{i,t}/X_{y,t-1}$, $\hat{D}_{i,t} \equiv D_{i,t}/X_{y,t-1}$, and $\hat{H}_{i,t} \equiv H_{i,t}/X_{y,t-1}$ such that the equilibrium conditions read:

- Utility

$$\hat{U}_t = \left((1 - \beta) + \beta \mathbb{E}_t \left[\left(\hat{U}_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right)^{\frac{1}{1-1/\psi}} \tag{15}$$

- Stochastic discount factor

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{\left(\hat{U}_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma}}{\mathbb{E}_t \left[\left(\hat{U}_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{16}$$

- Output

$$\hat{Y}_{i,t} = e^{(1-\alpha)(x_{yt}+z_{i,t})} \hat{K}_{i,t}^{\alpha} - f \hat{K}_{i,t} - \bar{f} \tag{17}$$

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- Capital accumulation

$$\hat{K}_{it+1}e^{x_{y,t}} = \hat{I}_{it} + (1 - \delta)\hat{K}_{it} \tag{18}$$

- Adjustment costs

$$\hat{H}_{i,t} = \frac{\theta_t}{2}\left(\frac{\hat{I}_{i,t}}{\hat{K}_{i,t}}\right)^2\hat{K}_{i,t} \tag{19}$$

- Firm value

$$\hat{V}_{i,t} = \max_{\chi_{i,t}}\left\{\max_{\hat{I}_{i,t}}\hat{D}_{i,t} + e^{x_{y,t}}\mathbb{E}_t\left[M_{t,t+1}\hat{V}_{i,t+1}\right], \kappa\right\} \tag{20}$$

$$\hat{D}_{i,t} = \hat{Y}_{i,t} - \hat{H}_{i,t} - \hat{I}_{i,t} \tag{21}$$

- Stock returns

$$R_{i,t+1} = \frac{\hat{V}_{i,t+1}}{\hat{V}_{i,t} - \hat{D}_{i,t}}e^{x_{y,t}} \tag{22}$$

4   Model Results

This section presents the calibration and the quantitative results of the model. Overall, our model can provide a quantitatively relevant account of the joint dynamics between uncertainty and risk premia dynamics at *low frequencies*, both the aggregate and in the cross-section of expected returns. We show that calibrating the stochastic processes for microeconomic and macroeconomic uncertainty to the data, our model can capture the low-frequency waves in the size premium, as well as the positive co-movement with the value premium, but a negative co-movement with the equity premium.

4.1 Calibration

Table 5 presents the monthly calibration of the benchmark model. Panel A reports the values for the preference parameters. The coefficient of relative risk aversion, γ, is set to 10 and the intertemporal elasticity of substitution, ψ, is set to 1.5, both of which are standard values in the long-run risks literature (e.g., Bansal and Yaron (2004)). The time discount factor is calibrated to be consistent with the level of the riskfree rate.

Panel B reports the calibration of the parameters relating to the production technology and adjustment costs. The curvature of the production function and depreciation rate of the capital stock are set to standard values in the production-based asset pricing literature (e.g., Jermann (1998)). The fixed cost parameter is calibrated to match the average book-to-market ratio. The adjustment cost parameters are calibrated to values from Bai, Hou, Kung, Li, and Zhang (2018).

Panel C reports the calibration of the stochastic processes. The consumption growth process is calibrated as in Bansal and Yaron (2004). The parameters governing the process for microeconomic uncertainty (TFP dispersion) is set to match the persistence and standard deviation of the low-frequency component of the empirical measure. Similarly, the parameters for the macroeconomic uncertainty process are set to match the persistence and standard deviation of the low-frequency component realized consumption growth volatility. We also calibrate the correlation between the two volatility processes to be consistent with the value estimated in the data. Overall, we calibrate the volatility process to capture the low-frequency patterns in the data documented in the section above.

4.2 Risk Premia

Table 6 reports the means and standard deviations of the equity premium, size premium, value premium, and riskfree rate. The model generates a sizable equity premium due to the exposure of aggregate dividend growth to long-run consumption risks. The representative investor has recursive preferences (with a preference for an early resolution of uncertainty), and is therefore strongly averse to long-run consumption uncertainty. Financial claims whose payoffs have high exposure to such low-frequency risks command high expected returns in financial markets. The low riskfree

rate is attributed to a strong precautionary savings motive arising from the significant long-run consumption uncertainty.

The model generates a significant size premium inline with the empirical counterpart. We sort firms by market cap into deciles, and form value-weighted portfolios in each decile. The mean of the returns in each decile from the model are reported in Table 2. The lowest decile is labeled as the *small* firms, while the highest decile is labeled as the *big* firms. The size effect is exhibited by the strong negative relation between market capitalization and expected returns. The size premium is computed as the average return to a long-short portfolio between the small and big portfolios.

The size premium arises due to the persistent mean-reverting idiosyncratic TFP shocks. Small market cap firms are characterized as having a recent history of bad idiosyncratic shocks and currently having a low stock of capital. Therefore, at portfolio formation, these firms are generating low revenues, but due to mean reversion, cash flows are expected to be larger in the future. As these firms have a low capital stock, they can do very little to hedge these shocks in immediate future by selling capital (and increasing payouts) in response to bad shocks. Consequently, the dividend payouts to shareholders will also be low currently (at portfolio formation), but expected to be larger in the future. These dynamics are illustrated in Figure 6, which shows the dynamics of small firms and big firms for the 12 months before, at, and the 12 months after portfolio formation. The diverging payout dynamics for small and big firms give rise to the size effect. Small (big) firms therefore have a larger (smaller) share of aggregate payouts in the short-term relative to the long-term. Consequently, small firms have higher exposure to long-run risks than big firms, consistent with evidence from Bansal, Dittmar, and Lundblad (2005). In equilibrium, small firms command higher expected returns than big firms.

The model also generates a sizable value premium that accords with the empirical analogue. Table 3 also reports the summary statistics for the book-to-market sorted portfolios, where the lowest decile are labeled as *growth* firms and the highest decile are labeled as the *value* firms. The model produces a strong positive relation between book-to-market and expected returns. The value premium is computed as the average return to a long-short portfolio between the value and growth portfolios.

The value premium arises due to the combination of the persistent mean-reverting shocks and

the investment frictions (asymmetric quadratic capital adjustment costs and proportional operating costs) as in Zhang (2005). Value firms are characterized by firms with a large capital stock, but have a recent history of bad idiosyncratic shocks prior to portfolio formation. Such firms have strong incentives to disinvest given the low marginal product of their capital. Absent investment frictions, value firms would liquidate a large portion of their unproductive capital, where the surplus funds would be used to increase the dividend payout. The partial irreversibilities, captured through the asymmetric quadratic adjustment costs, discourage large capital sales. The proportional operating costs reduce the funds available for payouts, especially for firms with large capital stocks. Consequently, value firms have low payouts at portfolio formation, however, due to mean reversion in the idiosyncratic shock, they are expected to have higher payouts in the future. These dynamics are depicted in Figure 7, which shows the dynamics of value firms and growth firms for the 12 months before, at, and the 12 months after portfolio formation. Value (growth) firms therefore have a larger (smaller) share of aggregate payouts in the short-term relative to the long-term. Consequently, value firms have higher exposure to long-run risks than growth firms. In equilibrium, value firms command higher expected returns than growth firms.

4.3   Low-Frequency Waves

The low-frequency fluctuations of the size and value premia in the model are driven by the volatility process for microeconomic uncertainty. In contrast, the low-frequency movements in the equity premium are driven by the persistent volatility process for macroeconomic uncertainty. In Figure 8 we show the dynamics of the idiosyncratic TFP process and the dividend yield for small and big firms 12 months before, at, and 12 months after portfolio formation conditional on high microeconomic and low microeconomic uncertainty at portfolio formation. During periods of high microeconomic uncertainty, the magnitude of the mean-reverting idiosyncratic shocks are larger, which magnifies the effects on cash flows. Consequently, dividend payouts for small firms are smaller (larger) in the short-term (long-term) compared to low microeconomic uncertainty periods. These payout dynamics imply that small firms have more exposure to long-run risks and big firms have lower exposure to long-run risks when microeconomic uncertainty is high (and big firms have lower). Therefore, the size premium is larger during periods of high microeconomic uncertainty, consistent with the

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data. In Table 7, we indeed verify the positive relation between microeconomic uncertainty and the size premium from the model. Importantly, in periods of low uncertainty, the size premium is not statistically significant, as in the data (e.g., the 1980-2000 period).

The value premium also increases during periods of high microeconomic uncertainty. Figure 8 shows the dynamics of the idiosyncratic TFP process and the dividend yield for value and growth firms 12 months before, at, and 12 months after portfolio formation conditional on high microeconomic and low microeconomic uncertainty at portfolio formation. The larger magnitude of the mean-reverting idiosyncratic shocks increases the exposure of value firms to long-run risks, while decreasing the exposure of growth firms to long-run risks. Consequently, the value premium is higher when microeconomic uncertainty is high, which is verified in Table 7. As in Bansal and Yaron (2004), when macroeconomic uncertainty about long-term growth is high, aggregate risk premia increases (and the riskfree rate declines) as the representative agent dislikes uncertainty about long-term consumption growth. These relations are reported in Table 7.

Table 4 compares the correlations, at low frequencies, between the model and the data. Overall, the model generates comovement patterns in risk premia and uncertainty that are consistent with the data. As discussed above, the dynamics of the size and value premia are driven by the stochastic process governing microeconomic uncertainty. Consequently, we find strong positive comovement between these three variables. The aggregate equity premium is driven by the dynamics of macroeconomic uncertainty, which is reflected in positive comovement between these variables. Given that we calibrate the processes for microeconomic and macroeconomic uncertainty to be negatively correlated, as in the data, we also get that the size and value premia are both negative related to the equity premium.

5   Conclusion

This paper examines the link between microeconomic uncertainty and the size premium across different frequencies in an investment model with heterogeneous firms. We document that the observed time-varying dispersion in firm-specific productivity can account for a large size premium in the 1960's and 1970's, the disappearance in the 1980's and 1990's, and reemergence in the 2000's.

Periods with a large (small) size premium coincide with high (low) microeconomic uncertainty. During episodes of high productivity dispersion, small firms increase their exposure to macroeconomic risks. Our model can also explain the strong positive low-frequency co-movement between size and value factors, but a negative relation with the market factor.

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Table 1: Size Premium Waves

This table reports average excess returns, volatility, and t-statistic of a size portfolios over different samples. We report these statistics for a portfolio that goes long low-market-equity (the 1st decile) stocks and short high-market-equity stocks (10th decile). The sample period is from July 1926 to December 2017 at a monthly frequency. Returns and volatilities are annualized.

	1926-1940 (1)	1941-1960 (2)	1961-1980 (3)	1981-2000 (4)	2001-2017 (5)
Avg. Return	11.23	9.43	9.12	-4.54	5.31
Volatility	49.55	20.06	17.37	17.93	12.39
t-statistic	0.86	2.10	2.35	-1.13	1.77

Table 2: Size-Sorted Portfolios

This table reports average excess returns of size-sorted portfolios. Stocks are sorted into 10 deciles based on market equity (NYSE breakpoints). In Panel A, we report the average excess return and respective t-statistics for each decile for the value-weighted portfolio. The sample period is from July 1926 to December 2017 at a monthly frequency. In Panel B, we report the model results for the value sorted portfolios.

	Small (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Big (10)	(1)-(10)
	Panel A: Data, Value-weighted										
Avg. Return	13.10	11.38	11.77	10.89	10.53	10.67	9.87	9.45	8.75	7.27	5.83
t-statistic	3.65	3.65	4.06	4.06	4.10	4.36	4.29	4.29	4.22	3.96	2.20
	Panel B: Model, Value-weighted										
Avg. Return	13.0	13.8	11.4	11.0	10.2	10.0	9.2	8.9	7.7	6.32	6.64

Table 3: Value-Sorted Portfolios

This table reports average excess returns of book-to-market-sorted portfolios. Stocks are sorted into 10 deciles based on book-to-market ratios (NYSE breakpoints). In Panel A, we report the average excess return and respective t-statistics for each decile for the value-weighted portfolio. The sample period is from July 1926 to December 2017 at a monthly frequency. In Panel B, we report the model results for the value sorted portfolios.

	Growth (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Value (10)	(10)-(1)
	Panel A: Data, Value-weighted										
Avg. Return	6.91	7.40	7.58	7.67	7.71	8.53	8.62	8.58	11.35	11.93	5.02
t-statistic	3.25	3.82	4.02	3.97	4.20	4.60	4.71	4.54	5.61	5.21	2.78
	Panel B: Model, Value-weighted										
Avg. Return	6.68	7.13	8.14	8.32	8.97	10.1	10.4	11.6	12.5	13.3	6.59

Table 4: Low-Frequency Comovement

This table reports the correlation between long-short size-sorted portfolio, long-short value-sorted portfolio, portfolio of market excess returns, long-short TFP-sorted portfolio, cross-sectional dispersion in TFP (micro uncertainty), aggregate volatility of GDP (macro uncertainty). Returns are cumulative over subsequent 10 years. We apply the Christiano and Fitzgerald (2003) band-pass filter to isolate frequencies between 20 and 50 years.

	Size (1)	Value (2)	Market (3)	TFP (4)	Micro (5)	Macro (6)
Panel A: Data, Low Frequency						
Size	1.00	0.66	−0.62	0.61	0.81	−0.83
Value		1.00	−0.50	0.44	0.22	−0.43
Market			1.00	−0.51	−0.64	0.76
TFP				1.00	0.64	−0.89
Micro Uncertainty					1.00	−0.72
Macro Uncertainty						1.00
Panel B: Model, Low Frequency						
Size	1.00	0.76	−0.24	0.89	0.71	−0.47
Value		1.00	−0.22	0.81	0.65	−0.39
Market			1.00	−0.35	−0.54	0.58
TFP				1.00	0.87	−0.53
Micro Uncertainty					1.00	−0.72
Macro Uncertainty						1.00

Table 5: Calibration

This table reports the parameter values used in the monthly calibration of the model. The table is divided into three categories: preferences, production, and dynamics parameters.

Parameter	Description	Model
A. Stochastic Discount Factor		
β	Time discount factor	0.998
γ	Relative risk aversion coefficient	10.0
ψ	Intertemporal elasticity of substitution	1.5
B. Firms		
α	Curvature parameter in the production function	0.30
δ	Rate of capital depreciation	0.01
ϕ	Leverage parameter	3.5
f	Proportional operating costs	0.5
$\bar{f}$	Fixed costs	0.7
θ^-	Downward convex capital adjustment costs	250
θ^+	Upward convex capital adjustment costs	150
$\bar{R}$	Delisting return	15%
C. Exogenous Dynamics		
$\bar{x}$	Mean of log aggregate consumption growth	1.9%/12
ρ_x	Persistence of x_t	0.985
σ_φ	Volatility parameter of x_t	0.044
ρ_z	Persistence of log firm-specific productivity	0.97
σ_x^L, σ_x^H	Volatility of log aggregate consumption	0.0063, 0.0084
σ_z^L, σ_z^H	Volatility of log firm-specific productivity	0.23, 0.30

Table 6: Financial Moments

	Data	Model
A. Mean		
Equity premium	7.96 %	7.64 %
Size premium	6.13 %	6.59 %
Value premium	5.73 %	5.47 %
Riskfree rate	1.80 %	1.76 %
B. Volatility		
Equity premium	18.51 %	21.6 %
Size premium	25.63 %	29.8 %
Value premium	22.15 %	20.7 %
Riskfree rate	3.00 %	1.76 %

Table 7: Conditional idiosyncratic and aggregate volatility

	mean [%]	
A. Idiosyncratic volatility	σ_z^L	σ_z^H
Size premium	2.91	10.5
t-statistic	(1.07)	(3.86)
Value premium	4.39	6.51
t-statistic	(2.32)	(3.44)
B. Aggregate volatility	σ_x^L	σ_x^H
Equity premium	6.23	9.05
Risk free rate	1.94	1.63

Figure 1: Size Premium and Microeconomic Uncertainty

This figure shows the time series returns of different size market strategies, along with two different measure of cross-sectional dispersion in TFP. The first size strategy we report is a portfolio that goes long low-market-equity (the 1st decile) stocks and short high-market-equity stocks (10th decile). We also report the return of a portfolio that go long stocks with high TFP shocks and short stocks with low TFP shocks, measure by the average TFP shocks in the previous three years. We report detrended cross-sectional dispersion in TFP, where TFP is from Compusat. We use the firm-level TFP data from İmrohoroğlu and Tüzel (2014). All series are standardize to have mean zero and variance one. We apply the Christiano and Fitzgerald (2003) band-pass filter to isolate frequencies between 20 and 50 years.
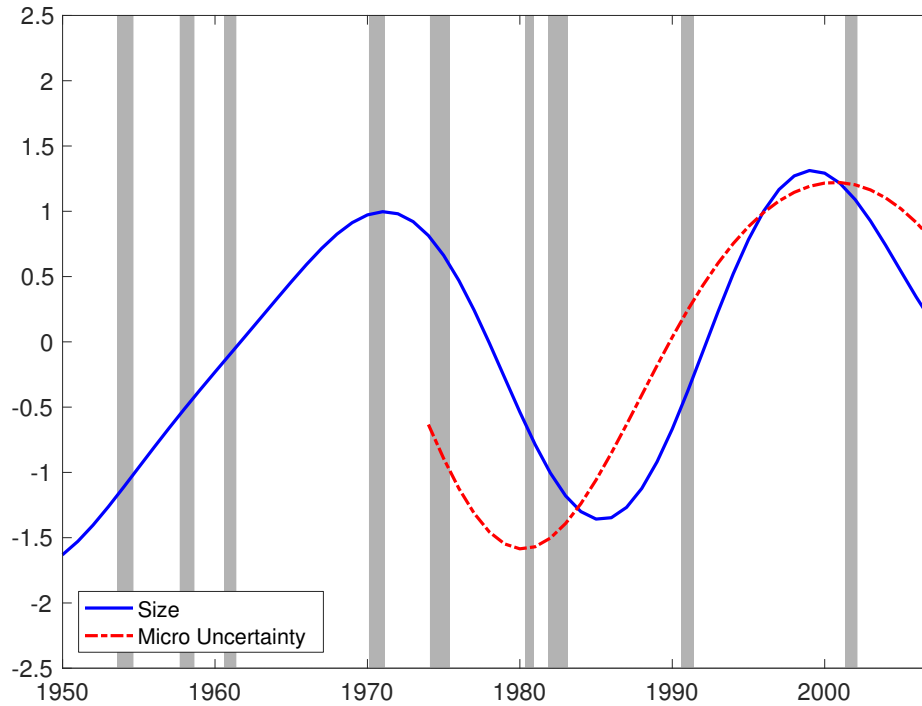
Figure 2: Size, Value, and Equity Premia
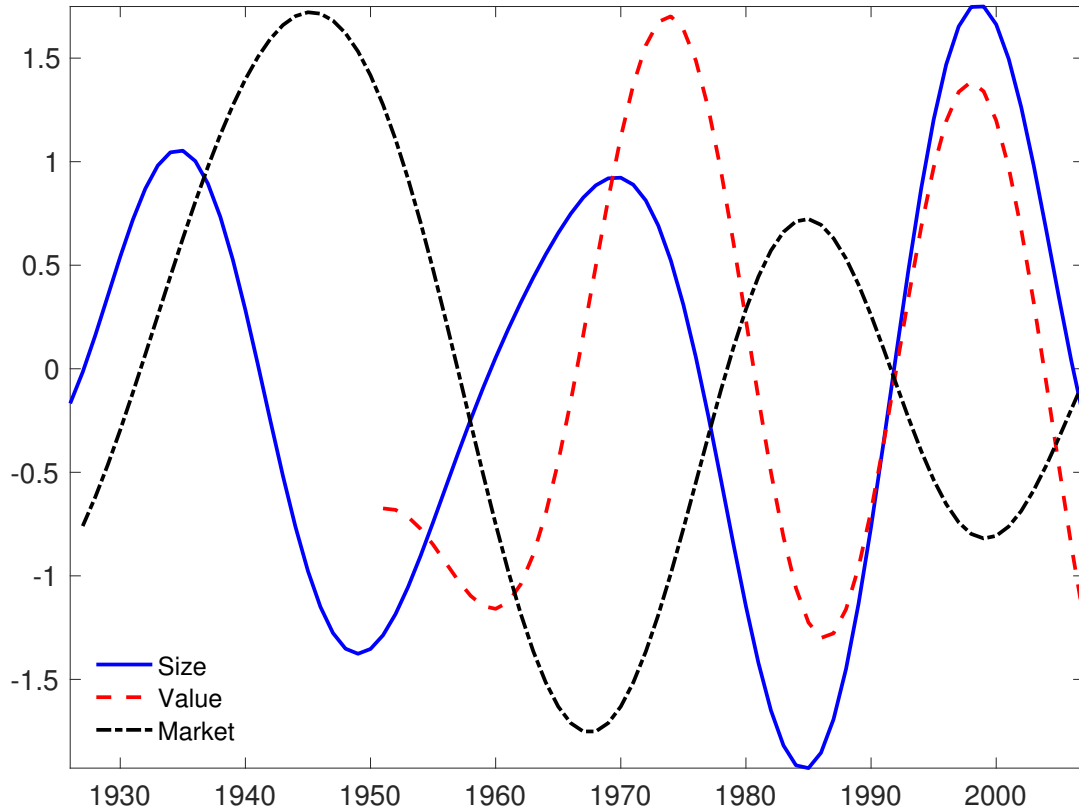
This figure shows the time series returns of size, value and market strategies. The value strategy is a portfolio that goes long high-book-to-market (the 10th decile) stocks and short low-book-to-market stocks (1st decile). The size strategy is a portfolio that goes long low-market-equity (the 1st decile) stocks and short high-market-equity stocks (10th decile). Finally, the market strategy is the market portfolio in excess of the risk-free rate of return. The sample period is from 1927 to to December 2017. We plot the annual series of the average cumulative return on the subsequent ten years. Returns are reported in percent per year. We apply the Christiano and Fitzgerald (2003) band-pass filter to isolate frequencies between 20 and 50 years.

Figure 3: Equity Premium and Macroeconomic Uncertainty

This figure shows the time series returns of market portfolio (10-year cumulative return) and different measure of aggregate volatility, include aggregate consumption, GDP and aggregate TFP volatility. We apply the Christiano and Fitzgerald (2003) band-pass filter to isolate frequencies between 20 and 50 years.
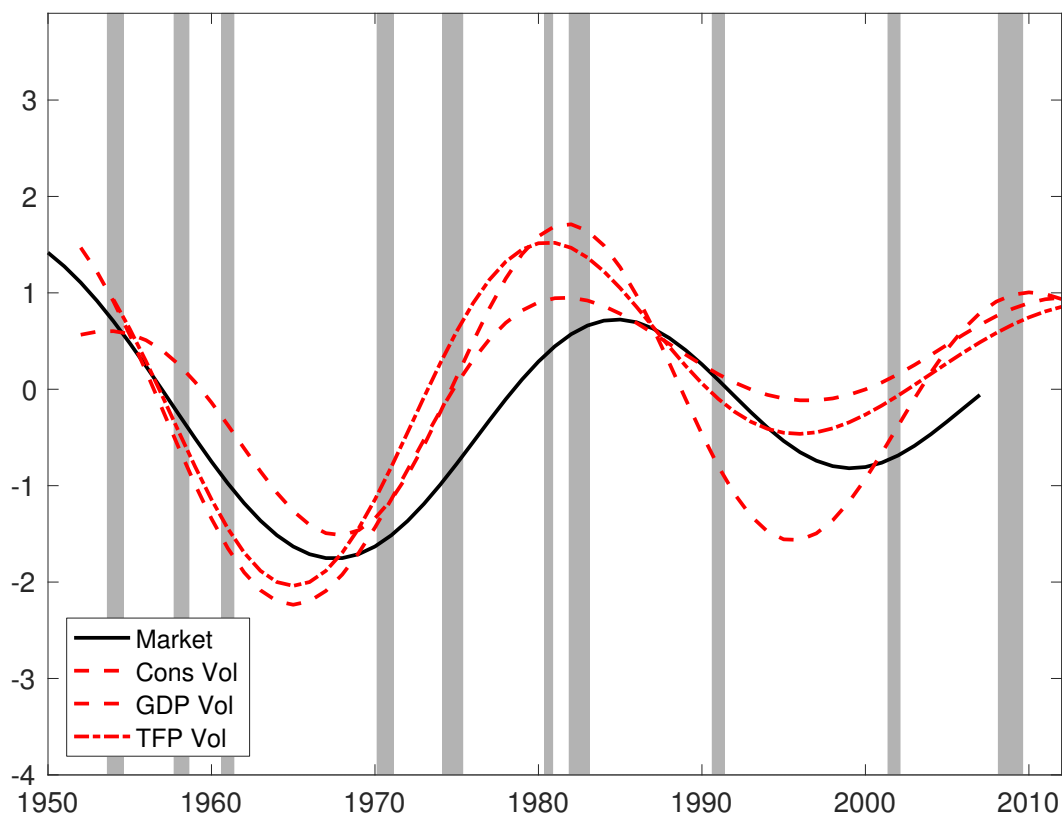
Figure 4: Micro and Macro Uncertainty Waves

This figure shows the time series returns of TFP dispersion and different measure of aggregate volatility, include aggregate consumption, GDP, and aggregate TFP volatility. We apply the Christiano and Fitzgerald (2003) band-pass filter to isolate frequencies between 20 and 100 years.
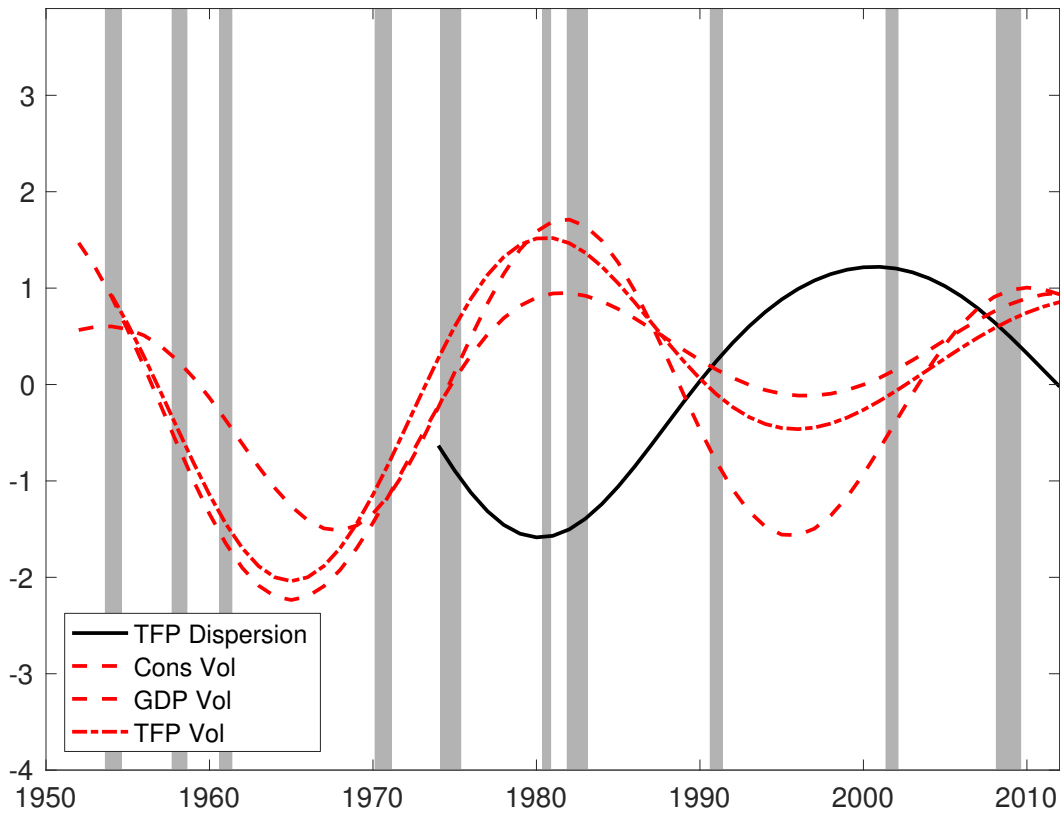
Figure 5: Size Premia with Different Size Measures

This figure shows the time series returns of different size market strategies. The first size strategy (ME-sorted) we report is a portfolio that goes long low-market-equity (the 1st decile) stocks and short high-market-equity stocks (10th decile). We also report size strategies based on market equity but excluding the month of January, book equity (BE), total assets (AT), and sales (SL). All series are standardize to have mean zero and variance one.
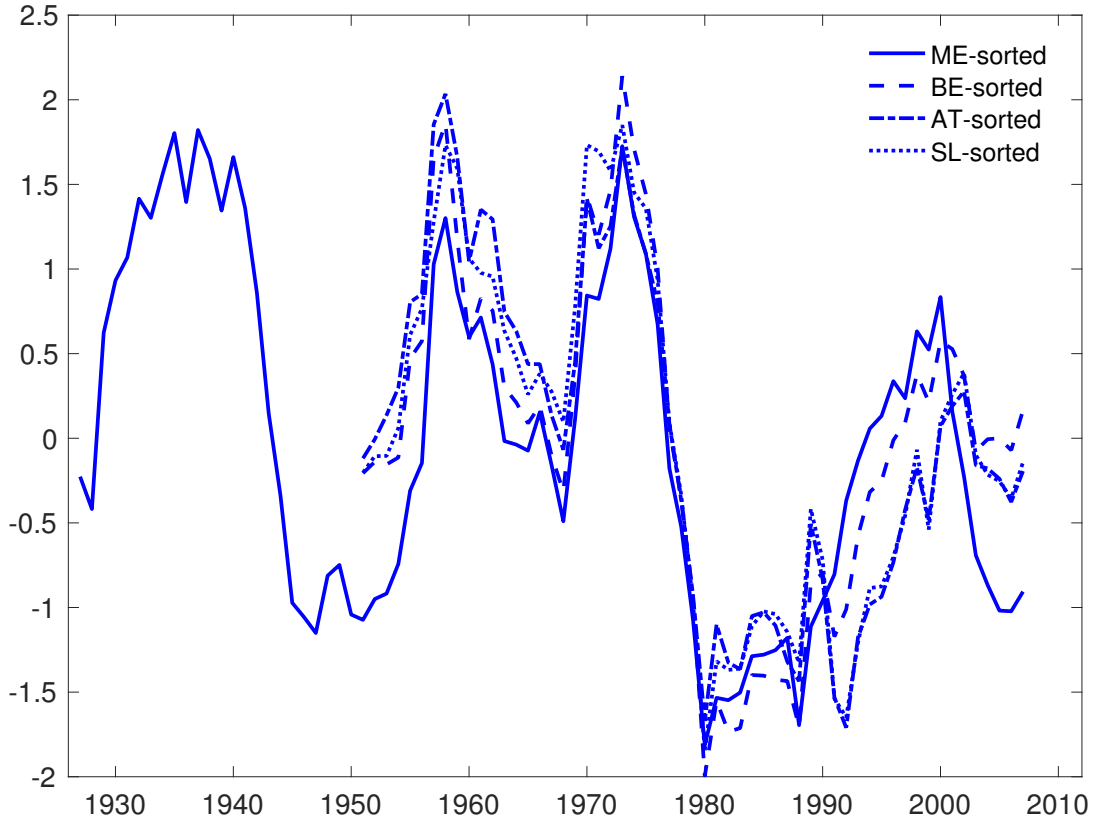
Figure 6: The Evolution of Small and Big Firms

These figures show the characteristics over time of small and big firms before and after portfolio formation.
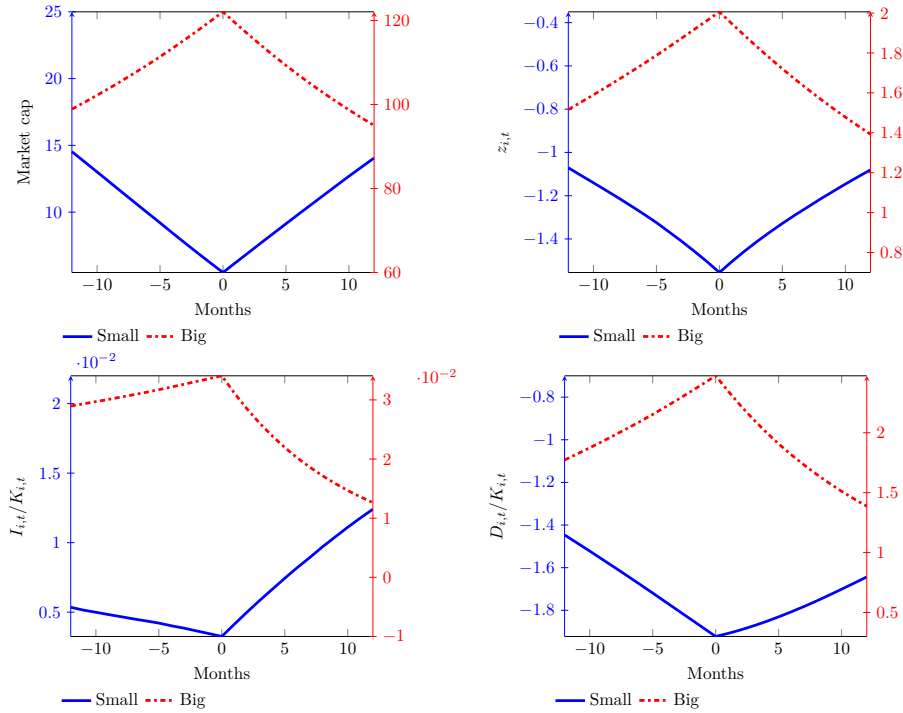

Figure 7: The Evolution of Growth and Value Firms

These figures show the characteristics over time of growth and value firms before and after portfolio formation.
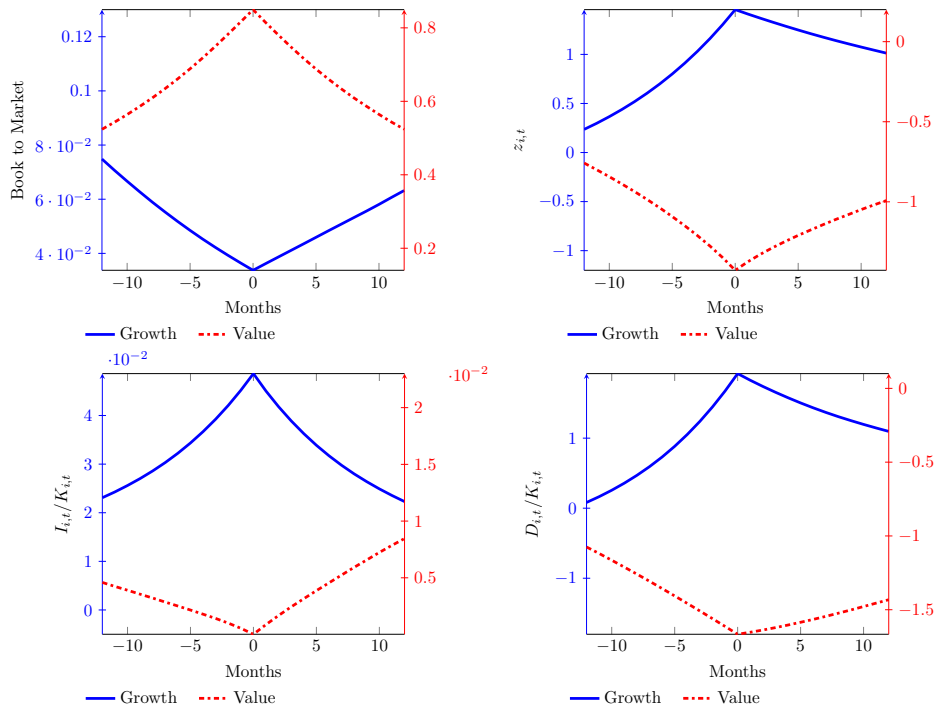
Figure 8: Conditional Firm Evolution

These figures show the characteristics over time of firms, conditional on the state of the firm-specific volatility.